CHAPTER

9

More Political Power

9.1 INTRODUCTION

We continue our study of political power in this chapter, beginning in **Sections 9.2** and **9.3** with two more quantitative measures of power. Both of these power indices were introduced in the late 1970s, the first appearing in Johnston (1978) and the second in Deegan–Packel (1978). These indices are similar in some ways to the Shapley–Shubik and Banzhaf indices introduced in Chapter 3, but they also differ in some important respects from these earlier ones as well as from each other. Additionally, we build on work of Brams, Affuso, and Kilgour (1989) in applying these indices to measure the power of the president in the context of the United States federal system as we did for the Shapley–Shubik index and the Banzhaf index in Chapter 3. It turns out, for example, that according to the Deegan–Packel index, the president has less than 1 percent of the power. The Johnston index, however, suggests that the president has 77 percent of the power.

In **Sections 9.4** and **9.5**, we offer a precise mathematical definition of what it means to say that two voters have comparable or incomparable power. This definition provides us with what is called an ordinal notion of power—"ordinal" referring to the fact that the ordering is not derived from the assignment of numbers, as it was with the four "cardinal" power indices (Shapley-Shubik, Banzhaf, Johnston, and Deegan-Packel). This ordinal notion of power is closely linked to the idea of "swap-robustness" from Chapter 2; the cognoscenti will recognize it as leading to the well-known "desirability relation on individuals." We conclude in **Section 9.6** with a theorem from Straffin (1980) that allows one to calculate the Shapley-Shubik index of a so-called "voting bloc" in a fairly trivial way.

9.2 THE JOHNSTON INDEX OF POWER

The Banzhaf index of power from Chapter 3 was based on the idea of a critical defection from a winning coalition. It does not, however, take into consideration the total number of players whose defection from a given coalition is critical. The point is, one might well argue that if a player p is the only one whose defection from C is critical, then this is a stronger indication of power than if, say, every player in C has a critical defection. This is the idea underlying the Johnston index of power as formalized in the following two definitions (which mimic those of **Section 3.4**).

DEFINITION. Suppose that p is one of the players in a yes–no voting system. Then the *total Johnston power* of p, denoted here by $\mathrm{TJP}(p)$, is the number arrived at as follows:

Suppose $C_1, \ldots, C_j$ are the winning coalitions for which p's defection is critical. Suppose n_1 is the number of players whose defection from C_1, is critical, n_2 is the number whose defection from C_2 is critical and so on up to n_j being the number of players whose defection from C_j is critical. Then

$$\mathrm{TJP}(P) = \frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_j}.$$

DEFINITION. Suppose that p_1 is a player in a yes–no voting system and that the other players are denoted by $p_2, p_3, \ldots, p_n$. Then the *Johnston index* of p_i, denoted here by $\mathrm{JI}(p_i)$, is the number given by

$$\mathrm{JI}(p_1) = \frac{\mathrm{TJP}(p_1)}{\mathrm{TJP}(p_1) + \cdots + \mathrm{TJP}(p_n)}.$$

Example:

Let's stick with the same three player example that we used to illustrate both the Shapley–Shubik index and the Banzhaf index in Chapter 3. Thus, p_1 has fifty votes, p_2 has forty-nine, and p_3 has one. Passage requires fifty-one votes. As before, the winning coalitions are

$$C_1 = \{p_1, p_2, p_3\}$$
$$C_2 = \{p_1, p_2\}$$
$$C_3 = \{p_1, p_3\}.$$

Now, p_1, has the only critical defection from C_1, but for both C_2 and C_3 it shares this property with the other member of the coalition. Thus, in calculating the absolute Johnston voting power of p_1, we get a contribution of 1 from p_1's presence in C_1, but only a contribution of $\frac{1}{2}$ each from its presence in C_2 and C_3. Similar comments hold for p_2 and p_3. Thus we have:

$$\text{TJP}(p_1) = 1 + \frac{1}{2} + \frac{1}{2} = 2$$
$$\text{TJP}(p_2) = 0 + \frac{1}{2} + 0 = \frac{1}{2}$$
$$\text{TJP}(p_3) = 0 + 0 + \frac{1}{2} = \frac{1}{2}$$

and

$$\text{JI}(p_1) = \frac{2}{2 + \frac{1}{2} + \frac{1}{2}} = \frac{2}{3}$$
$$\text{JI}(p_2) = \frac{\frac{1}{2}}{2 + \frac{1}{2} + \frac{1}{2}} = \frac{1}{6}$$
$$\text{JI}(p_3) = \frac{\frac{1}{2}}{2 + \frac{1}{2} + \frac{1}{2}} = \frac{1}{6}.$$

Notice that these turn out to agree with the Shapley–Shubik values in **Section 3.2**.

For yes–no voting systems where the number of winning coalitions is reasonably small, one can calculate total Johnston power by using

a chart as we did for total Banzhaf power in **Section 3.6**. The difference is that, with Johnston power, one needs to identify which voters have critical defections from which winning coalitions. We illustrate this by calculating the total Johnston power for the European Economic Community as set up in 1958. Critical defections are italicized in the chart below, and the number of countries italicized in a winning coalition determines the fractions that occur to the right of that coalition. Thus, in the winning coalition listed below as *FIBN*L, defections by the four countries F, I, B, and N are critical. Hence, each of these four countries receives a contribution of $\frac{1}{4}$ from this winning coalition towards its total Johnston power.

	F	**G**	**I**	**B**	**N**	**L**
FGI	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$			
FGBN	$\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$\frac{1}{4}$	
FIBN	$\frac{1}{4}$		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
GIBN		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
*FGI*L	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$			
*FGBN*L	$\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$\frac{1}{4}$	
*FIBN*L	$\frac{1}{4}$		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
*GIBN*L		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
*FGI*B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$			
*FGI*N	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$			
*FGI*BL	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$			
*FGI*NL	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$			
FGIBN						
FGIBNL						
TJP	3	3	3	$\frac{3}{2}$	$\frac{3}{2}$	0
JI	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	0

Summarizing these results as we did for the previous power indices in Chapter 3 yields the following:

Country	Votes	Percentage of votes	JI	Percentage of power
France	4	23.5	$\frac{1}{4}$	25.0
Germany	4	23.5	$\frac{1}{4}$	25.0
Italy	4	23.5	$\frac{1}{4}$	25.0
Belgium	2	11.8	$\frac{1}{8}$	12.5
Netherlands	2	11.8	$\frac{1}{8}$	12.5
Luxembourg	1	5.9	0	0

The Johnston Index of the President

To calculate the Johnston index of the president, we need a breakdown of the types of winning coalitions possible since we also must worry about how many players in each coalition have defections that are critical. In the chart that follows, we will give a name (like "T_{11}") for each type of winning coalition, and we will describe them by an expression like "67 S and 290 H" to indicate that this type of coalition is made up of 67 senators and 290 members of the House. "P" stands for "president."

Description of the winning coalitions	Number of critical defections	Whose defection is critical
T_{11}: 67 S and 290 H	357	S H
T_{12}: 67 S and 291–435 H	67	S
T_{13}: 68–100 S and 290 H	290	H
T_{21}: P and 51 S and 218 H	270	P S H
T_{22}: P and 51 S and 219–289 H	52	P S
T_{23}: P and 52–66 S and 218 H	219	P H
T_{24}: P and 52–66 S and 219–289 H	1	P
T_{31}: P and 67–100 S and 218 H	219	P H
T_{32}: P and 67–100 S and 219–289 H	1	P
T_{41}: P and 51 S and 290–435 H	52	P S
T_{42}: P and 52–66 S and 290–435 H	1	P

Now we can calculate the total Johnston power of the president, a member of the Senate, and a member of the House.

The winning coalitions involved in calculating the total Johnston power of the president can be obtained from column three above. They are

$$T_{21}, T_{22}, T_{23}, T_{24}, T_{31}, T_{32}, T_{41}, T_{42}.$$

Let $|T_{21}|$ denote the number of coalitions of type T_{21}. If we were calculating the total Banzhaf power of the president, we would just add the numbers: $|T_{21}|, |T_{22}|$, etc. However, since we are calculating the Johnston power, we must "adjust" each factor by dividing it by the number of people whose defection from such a coalition is critical. These can be obtained from column two above. Thus,

$$\text{TJP (The President)} = \frac{1}{270}|T_{21}| + \frac{1}{52}|T_{22}| + \frac{1}{219}|T_{23}| + |T_{24}| + \frac{1}{219}|T_{31}| + |T_{32}| + \frac{1}{52}|T_{41}| + |T_{42}|.$$

The calculation of, say, $|T_{24}|$, proceeds in a way similar to what we did in the calculations of the Banzhaf power. That is,

$$|T_{24}| = \left[\binom{100}{52} + \cdots + \binom{100}{66}\right] \times \left[\binom{435}{219} + \cdots + \binom{435}{289}\right].$$

Expressing $|T_{21}|, |T_{23}|$, etc. in "n choose k" notation is left to the reader. (See Exercises 5 and 6 at the end of the chapter.)

Now let's consider a fixed member of the Senate. Column three of the previous chart again shows which types of coalitions will have to be considered. They are

$$T_{11}, T_{12}, T_{21}, T_{22}, T_{41}.$$

Here, however, we have to be a little careful since we do not want to use, for example, $|T_{11}|$. The point is, lots of the coalitions of type T_{11} do not even include as a member the particular senator we are considering. Thus, the contribution from type T_{11} coalitions will involve the number of ways of choosing 66 other senators from the pool of 99 remaining senators multiplied by the number of ways of choosing 290 members of the House from the available 435. In particular, the "n choose k" expression will involve

$$\binom{99}{66} \text{ and not } \binom{100}{67}$$

With this potential pitfall confronted, the calculations proceed in a way similar to what we've done before, yielding

$$\begin{aligned}\text{TJP(A Senator)} = {} & \frac{1}{357}\left[\binom{99}{66} \times \binom{435}{290}\right] \\ & + \frac{1}{67}\binom{99}{66}\left[\binom{435}{291} + \cdots + \binom{435}{435}\right] \\ & + \frac{1}{270}\left[\binom{99}{50} \times \binom{435}{218}\right] \\ & + \frac{1}{52}\binom{99}{50}\left[\binom{435}{219} + \cdots + \binom{435}{289}\right] \\ & + \frac{1}{52}\binom{99}{50}\left[\binom{435}{290} + \cdots + \binom{435}{435}\right].\end{aligned}$$

Expressing the total Johnston power of a member of the House of Representatives in a similar fashion is left to the reader. (See Exercise 8 at the end of the chapter.)

To obtain the desired Johnston indices, we sum the total Johnston power of the 536 players involved, and then divide by the total. The results turn out to be:

JI(The president)	= .77
JI(A senator)	= .0016
JI(A member of the House)	= .00017.

Expressing these in terms of percentages of power instead of small decimals yields:

(Johnston) Power held by the president	= 77%
(Johnston) Power held by the Senate	= 16%
(Johnston) Power held by the House	= 7%

The striking thing to notice is the very different measure of power assigned the president by this index as opposed to those in Chapter 3. A little more on this will be said in the concluding section of this chapter, but there is no substitute for going directly to the literature.

9.3 THE DEEGAN–PACKEL INDEX OF POWER

In 1978, Deegan and Packel introduced a power index based on three assumptions:

1. Only minimal winning coalitions should be considered when determining the relative power of voters.
2. All minimal winning coalitions form with equal probability.
3. The amount or power a player derives from belonging to some minimal winning coalition is the same as that derived by any other player belonging to that same minimal winning coalition.

These assumptions, in fact, uniquely determine a power index. Moreover, the calculation required involves a nice blend of what we did with the two procedures for Banzhaf power (**Section 3.5**) and the calculation of Johnston power (**Section 9.2**). We begin with two definitions.

DEFINITION. Suppose that p is one of the voters in a yes–voting system. Then the *total Deegan–Packel power*of p, denoted here by TDPP(p), is the number arrived at as follows:

Suppose $C_1, \ldots, C_j$ are the *minimal* winning coalitions to which p belongs. Suppose n_1 is the number of voters in C_1, n_2 is the number in C_2, and so on up to n_j being the number of voters in C_j. Then

$$\text{TDPP}(p) = \frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_j}.$$