

MA 665 EXERCISES 1

- (1) Let R be a ring and M an R -module. An element $m \in M$ is called a *torsion element* if $rm = 0$ for some nonzero $r \in R$. Prove that if R is an integral domain, then the set of torsion elements in M is a submodule of M . Give an example where R is not an integral domain, and the set of torsion elements in M is not a submodule of M .
- (2) Let R be a commutative ring with unit. Prove that M and $\text{Hom}_R(R, M)$ are isomorphic as R -modules.
- (3) Let R be a commutative ring and let A, B, M be R -modules. Prove that
- $$\text{Hom}_R(A \times B, M) \cong \text{Hom}_R(A, M) \times \text{Hom}_R(B, M)$$
- and
- $$\text{Hom}_R(M, A \times B) \cong \text{Hom}_R(M, A) \times \text{Hom}_R(M, B).$$