A Day of Definitions

Basic Building Blocks

Segments, Lines and Polygons

Given two points A and B ∃! line containing them, $\overleftrightarrow{AB}$. Why?

We can identify A with the number 0 and B with any positive real number. Why?

Why are there infinitely many points on $\overleftrightarrow{AB}$?

Distance

$d(P,Q) = |x_P - x_Q|$ Where do we get this definition?

Can there be two different distances associated with two points? Why not?

Notation: $d(P,Q) \equiv PQ$

Betweenness

C lies between A and B if A,B,C distinct points on the same line and AC + CB = AB.

Notation: A*C*B means C lies between A and B

Given two points A and B the line segment $\overline{AB}$ consists of A, B, and all points that lie between A and B.

A, B = endpoints all other points = interior

AB = length of segment

Plane Figures

A figure in the plane is a set of points in the plane.

Convex: it contains all interior points of all lines segments joining any two points.

Non convex = concave

Rays and Angles

For $A \neq B$, the ray $\overrightarrow{AB}$ is

$$\overrightarrow{AB} = \overline{AB} \cup \{D \in \overleftrightarrow{AB} \mid A * B * D\}$$

$\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite if $A*B*C$.

Rays and Angles

Two rays emanating from the same point form two angles.

Common endpoint = vertex

Rays = sides

If the rays coincide we have one angle of measure 0 and another of measure 360.

Rays and Angles

If union of the rays is a straight line, each angle has measure 180. Called a straight angle.

For all others, one angle has a unique measure between 0 and 180.

If $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the rays the angle is $\angle ABC$ and it measure is $m\angle ABC$

Rays and Angles

Rays divide plane into 2 sets

Interior:

Exterior:

Are the sides of the angle in either set?

Rays and Angles

Acute

Obtuse

Right

Complimentary

Supplementary

Rays and Angles

What is difference between supplementary angles and angles that form a linear pair?

Polygons

Let $A_1, A_2, \ldots, A_n$ be distinct points in plane so that no three consecutive points are collinear

Suppose that no two of the segments

$$\overline{A_1A_2}, \overline{A_2A_3}, \ldots, \overline{A_{n-1}A_n}, \overline{A_nA_1}$$

share an interior point

The n-gon is $P_n = \overline{A_1A_2} \cup \overline{A_2A_3} \cup \ldots \cup \overline{A_{n-1}A_n} \cup \overline{A_nA_1}$

Points = vertices segments = sides

Polygons

Polygon divides plane into two sets: interior and exterior

How do we define the interior?

If interior is convex, polygon is called convex.

Regular – all sides are congruent and all angles are congruent

Polygons

3-gon = triangle

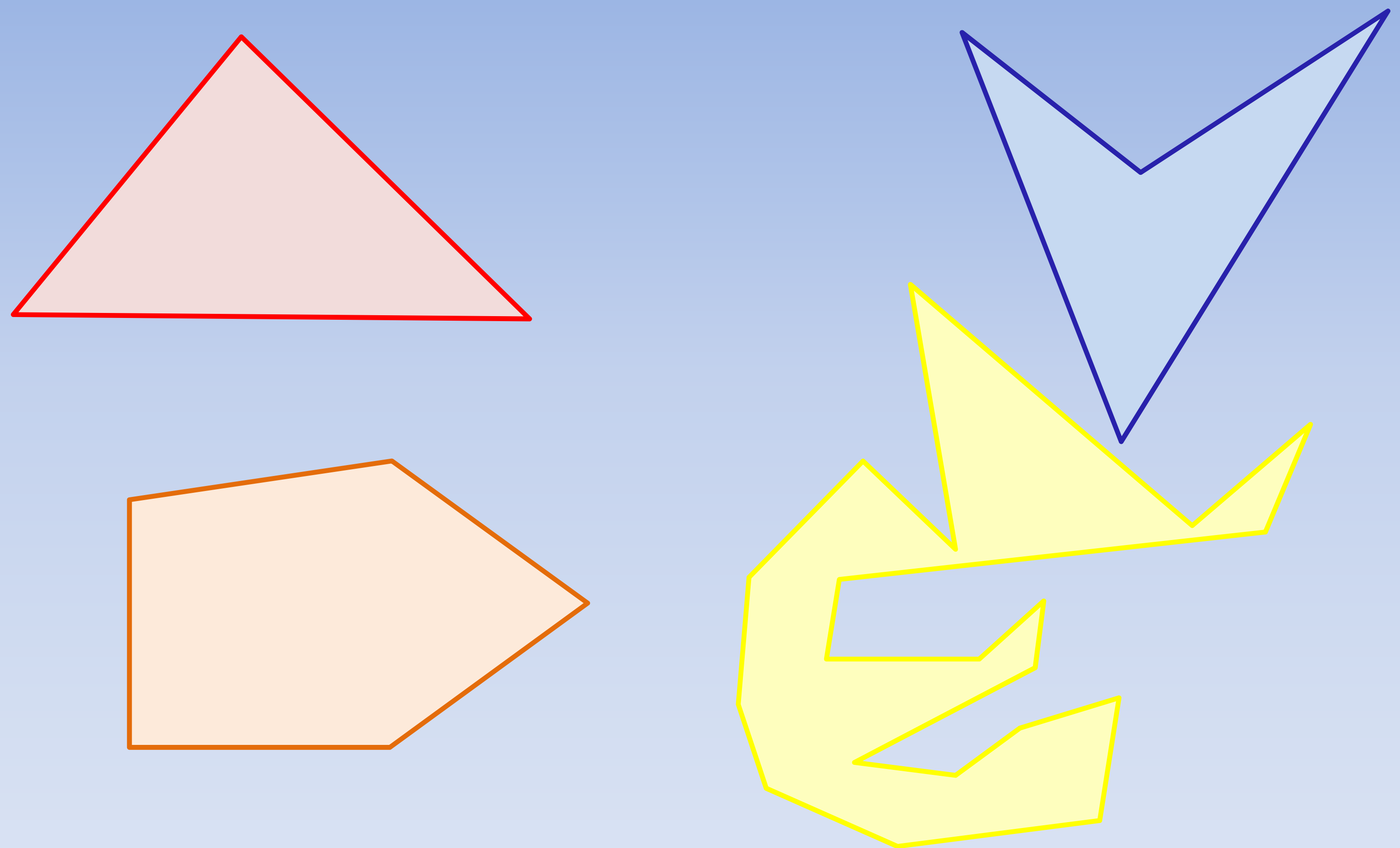
4-gon = quadrilateral

5-gon = pentagon

6-gon = hexagon

heptagon, octagon, nonagon, decagon, undecagon, dodecagon, etcagon

Examples

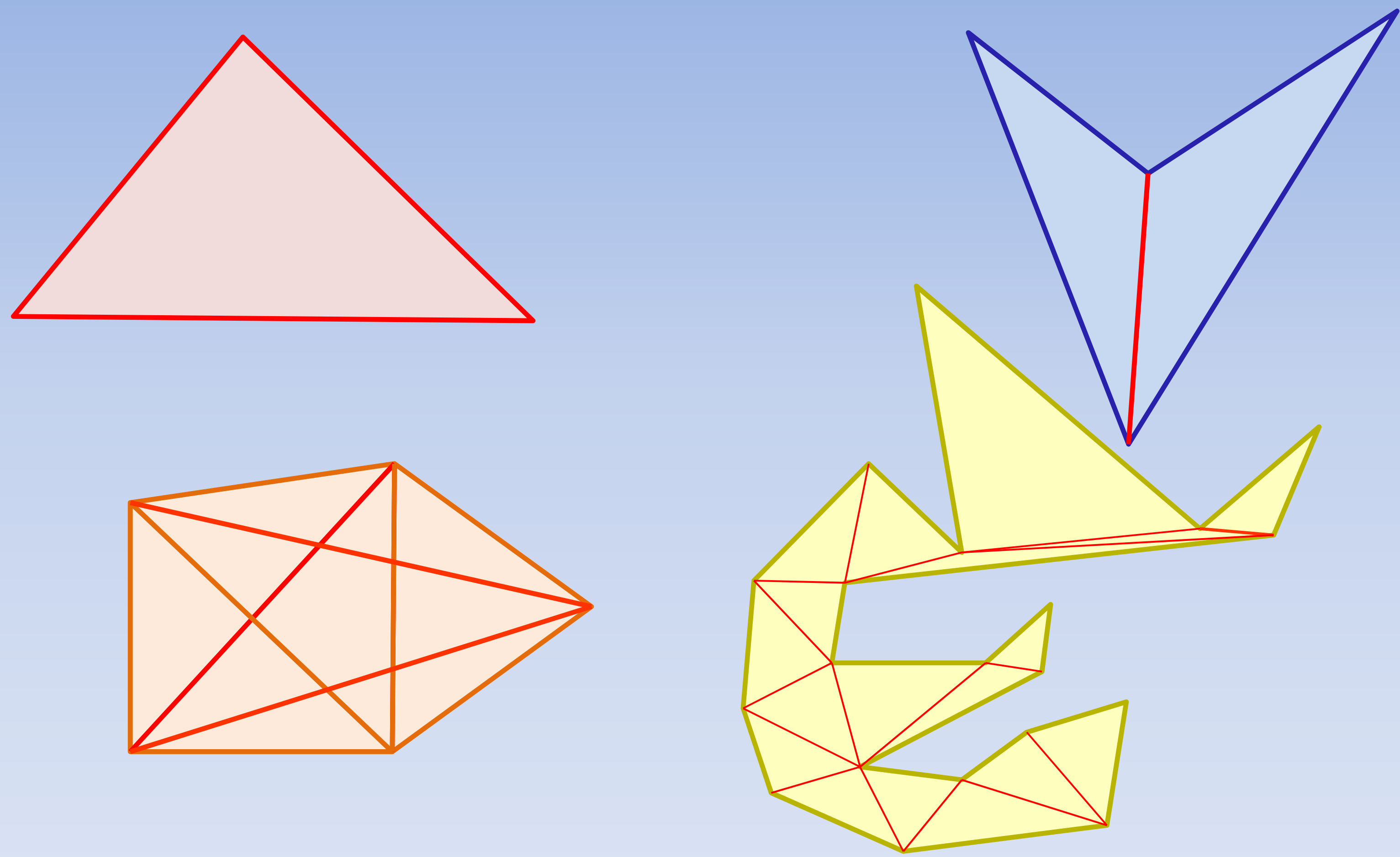


Statement

Every polygon can be written as a union of triangles that share only vertices and sides.

True or False

Examples



Similarity

1. Definition: Two triangles are similar if their corresponding angles are equal.
2. Definition: Triangles are similar if they have the same shape, but can be different sizes.
3. Definition: Two geometric shapes are similar if there is a rigid motion of the plane that maps one onto the other.

Definition

We need to be more precise, more inclusive.

Working Definition: Two figures, P and P′, are <u>similar</u> if there exists a positive real number k and an onto function $f: P \to P'$ so that for all A,B in P $f(A)f(B) = A'B' = k\ AB$.

k = coefficient of similarity

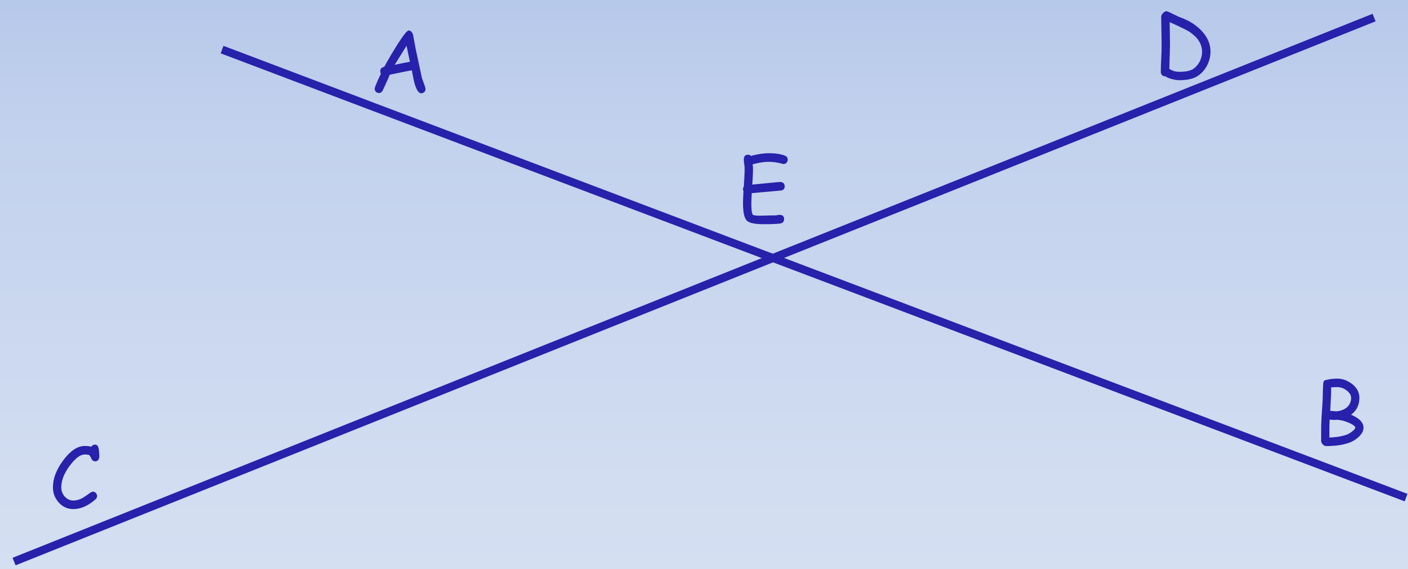
If k = 1, we say that the figures are congruent.

Similarity Facts

- Two lines are congruent
- Two rays are congruent
- Any two segments are similar
- Two segments are congruent iff they have the same length
- Any two circles are similar.
- Two circles are congruent iff they have equal radii
- Two angles are congruent iff they have equal measures.

Vertical Angles

If two distinct lines intersect they form 4 angles having a common vertex.



Which are vertical angles?

THEOREM

Theorem 1: Vertical angles are congruent.

Proof:

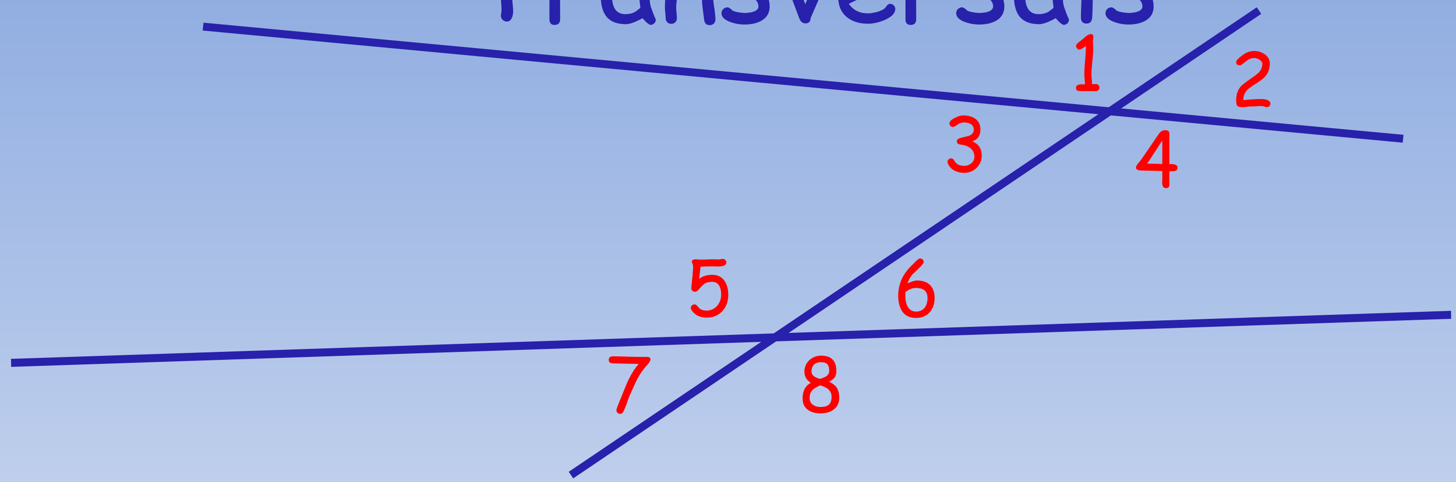
Lines

Two lines are parallel if they do not intersect.

Do we want a line to be parallel to itself?

If two lines are not parallel they intersect in a unique point. Why? Does your answer above affect this?

Transversals



Corresponding angles:

Alternate interior angles:

Alternate exterior angles:

Same side interior angles:

THEOREM

<u>Theorem 2:</u> Let l and m be distinct lines and let t be a transversal. The following are equivalent. (TFAE)

(1) l and m are parallel.

(2) Any two corresponding angles are congruent.

(3) Any two alternate interior angles are congruent.

(4) Any two alternate exterior angles are congruent.

(5) Any two same side interior angles are supplementary.

Proof

We will show that $1 \Rightarrow 3$ and $3 \Rightarrow 1$.

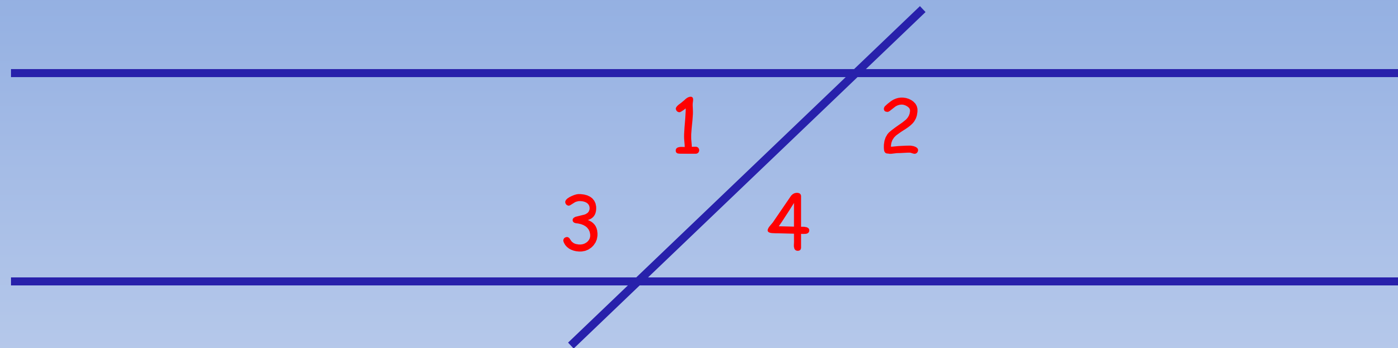
To complete this proof we could show either: $1 \Leftrightarrow 2$, $1 \Leftrightarrow 3$, $1 \Leftrightarrow 4$, $1 \Leftrightarrow 5$

OR: $1 \Leftrightarrow 2 \Leftrightarrow 3 \Leftrightarrow 4 \Leftrightarrow 5$

OR: $1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 4 \Rightarrow 5 \Rightarrow 1$

OR: $1 \Rightarrow 3 \Rightarrow 4 \Rightarrow 5 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1$

Proof: $1 \Rightarrow 3$



(Set up proof by contradiction).

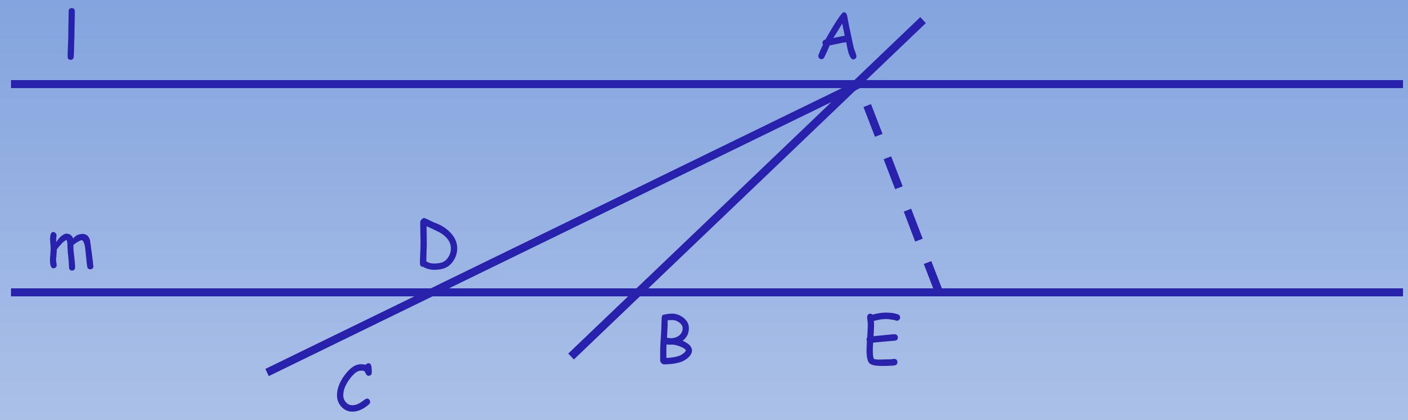
Assume $l \parallel m$ and $\angle 1 \neq \angle 4$.

We know

$$m\angle 1 + m\angle 2 = 180$$

and

$$m\angle 3 + m\angle 4 = 180.$$

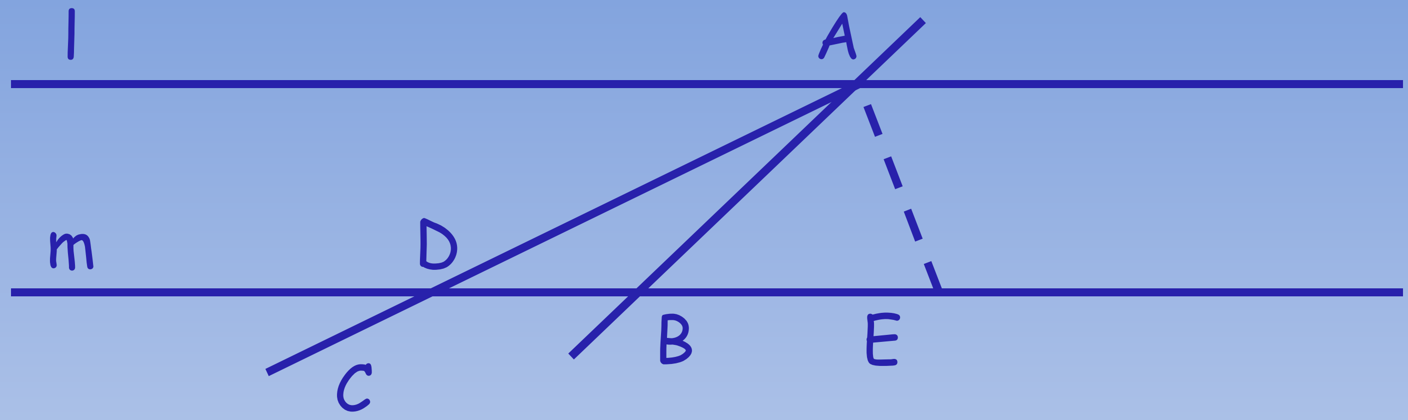


Since $\angle 1 \neq \angle 4$, we may assume $m\angle 1 > m\angle 4$. (Why?)

$\exists$ ray AC on opposite side of t from $\angle 4$ so that $m\angle CAB = m\angle ABE$.

Let $m \cap AC = D$.

$\exists$ E in m on opposite side of t from D so that $AD = BE$.



Then, AD = BE, $m\angle DAB = m\angle EBA$, and AB=AB.

Therefore by SAS $\Delta DAB \cong \Delta EBA$

$\Rightarrow m\angle DBA = m\angle 3 = m\angle BAE$

$\Rightarrow \angle DAB$ and $\angle BAE$ form linear pair

$\Rightarrow$ A, D, E collinear

$\Rightarrow$ A lies on m

$\Rightarrow$ l and m not parallel.

Thus we now have that

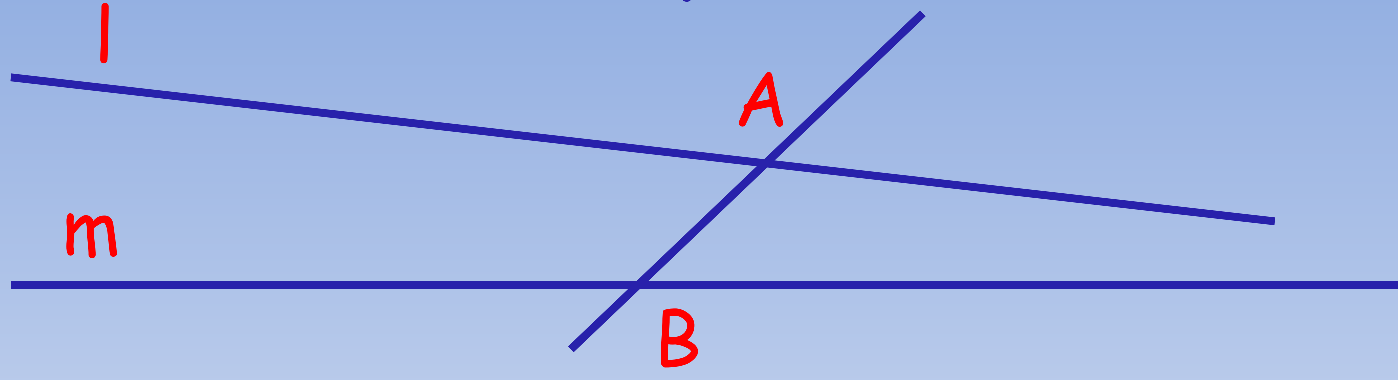
l and m not parallel

AND we are given that

l and m are parallel.

In other words we have $R \wedge \sim R$, a contradiction. Thus, $1 \wedge \sim 3$ leads to a contradiction so $1 \Rightarrow 3$

Proof: 3 $\Rightarrow$ 1

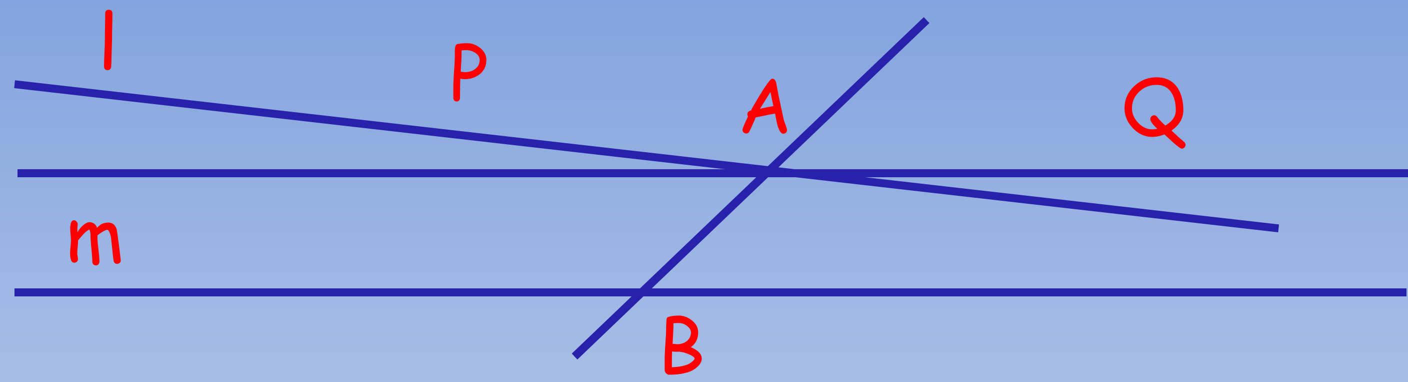


(Proof by Contrapositive).

[Assume ~1 and we need to deduce ~3.]

Assume l and m not parallel.

Let A and B be intersection of t with l and m.



P,Q on l so that P * A * Q

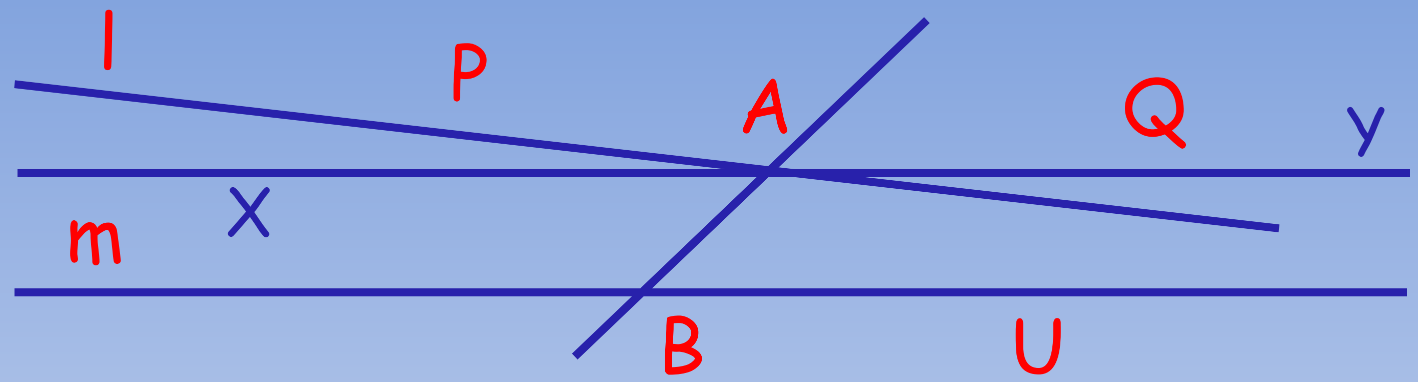
Let R be on m on same side of t as P

Let S be on m on same side of t as Q

$\exists$ line n through A parallel to m

Choose X,Y on n so that X * A * Y

n $\neq$ l, so we may assume n is interior to $\angle$PAB.



Thus, $m\angle PAX > 0$.

By first part of proof,

$m\angle BAX = m\angle UBA = m\angle 4$

Thus,

$m\angle 1 = m\angle PAB = m\angle PAX + m\angle XAB >$
$m\angle UBA = m\angle 4$.

Thus, $m\angle 1 \neq m\angle 4$

Thus we now have that $\sim 1 \Rightarrow \sim 3$ which is logically equivalent to $3 \Rightarrow 1$.