

MA 341
Topics in Geometry
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Spotted on the back of a t-shirt worn by
LAPD Bomb Squad:

**If you see me running,
try to keep up!**

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Syllabus and Course Outline
<http://www.ms.uky.edu/~droyster/courses/fall11/MA341>
BlackBoard: <http://elearning.uky.edu>
3 tests & final
Weekly homework
Assignments are all on Blackboard to be downloaded.
Software: Geogebra (<http://www.geogebra.org>)
Free and available for Windows, Mac, Linux

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A Short History of Geometry

Geometry - a collection of empirically discovered principles about lengths, angles, areas, and volumes developed to meet practical need in surveying, construction, astronomy, navigation

Earliest records traced to early peoples in the ancient Indus Valley (Harappan Civilization), and ancient Babylonia from around 3000 BCE.

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A Short History of Geometry



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A Short History of Geometry

- The **Indus Valley Civilization** - a Bronze Age civilization (3300-1300 BCE; mature period 2600-1900 BCE)
- centered mostly in the western part of the Indian Subcontinent.
- Primarily centered along the Indus and the Punjab region, the civilization extended into most of what is now Pakistan, as well as extending into the westernmost states of modern-day India, southeastern Afghanistan and the easternmost part of Balochistan, Iran.
- The mature phase of this civilization is known as the **Harappan Civilization**, as the first of its cities to be unearthed was the one at Harappa,

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Babylonian Geometry

Some general rules for measuring areas and volumes.

Circumference of a circle = 3 times the diameter
area as one-twelfth the square of the circumference

The volume of a cylinder = product of base & height

Knew a type of the Pythagorean theorem

Knew of theorems on the ratios of the sides of similar triangles

Had a trigonometry table

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Egyptian Geometry

Egyptians had a correct formula for the volume of a frustum of a square pyramid

Area of Circle $\approx [(\text{Diameter}) \times 8/9]^2$.

Makes π is $4 \times (8/9)^2$ (or 3.160493...) with an error of slightly over 0.63 percent.

Was not surpassed until Archimedes' approximation of $211875/67441 = 3.14163$, which had an error of just over 1 in 10,000

Also knew a type of Pythagorean theorem

Extremely accurate in construction, making the right angles in the Great Pyramid of Giza accurate to one part in 27,000

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Early Chinese Geometry

Zhoubi suanjing (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven) (c. 100 BCE-c. 100 AD)

- States and uses Pythagorean theorem for surveying, astronomy, etc. Also a proof of Pythagorean theorem.

The Nine Chapters on the Mathematical Art (Jiuzhang Suanshu) (c. 100 BCE-50 AD)

- areas of plane figures, square, rectangle, triangle, trapezoid, circle, circle segment, sphere segment, annulus - some accurate, some approximations.
- Construction consultations: volumes of cube, rectangular parallelepiped, prism frustums, pyramid, triangular pyramid, tetrahedron, cylinder, cone, and conic frustum, sphere -- some approximations, some use $\pi=3$
- Right triangles: applications of Pythagorean theorem and similar triangles

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Early Chinese Geometry

- Liu Hui (c. 263)
 - Approximates π by approximating circles polygons, doubling the number of sides to get better approximations. From 96 and 192 sided polygons, he approximates π as 3.141014 and suggested 3.14 as a practical approx.
 - States principle of exhaustion for circles
- Zu Chongzhi (429-500) Astronomer, mathematician, engineer.
 - Determined π to 7 digits: 3.1415926. Recommended use 355/113 for close approx. and 22/7 for rough approx.
 - Found accurate formula for volume of a sphere.

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Vedic Geometry

- The *Satapatha Brahmana* (9th century BCE) contains rules for ritual geometric constructions that are similar to the *Sulba Sutras*.
- The *Sulba Sūtras* (c. 700-400 BCE) list rules for the construction of sacrificial fire altars leading to the Pythagorean theorem.
- Bakhshali manuscript contains some geometric problems about volumes of irregular solids
- Aryabhata's *Aryabhatiya* (499 AD) includes the computation of areas and volumes.
- Brahmagupta wrote his work *Brāhma Sphuṭa Siddhānta* in 628 CE included more geometric areas

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Vedic Geometry

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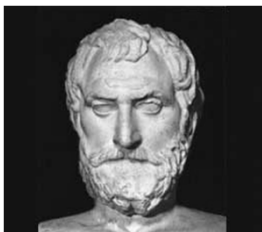
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Early Greek Geometers

Thales of Miletus
(624-547 BCE)

- A circle is bisected by any diameter.
- The base angles of an isosceles triangle are equal.
- The angles between two intersecting straight lines are equal.
- Two triangles are congruent if they have two angles and one side equal.
- An angle inscribed in a semicircle is a right angle.



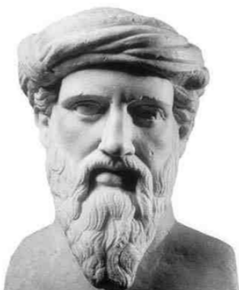
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Pythagoras of Samos (569-475 BC)

1. First to deduce logically geometric facts from basic principles
2. angles of a triangle sum to 180
3. *Pythagorean Theorem* for a right-angled triangles



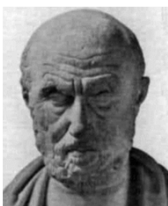
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Hippocrates of Chios (470-410 BCE)

Geometric solutions to quadratic equations
Early methods of integration
Studied classic problem of squaring the circle showing how to square a *lune*.
Worked on duplicating the cube which he showed equivalent to constructing two mean proportionals between a number and its double.
First to show that ratio of areas of two circles was equal to the ratio of the squares of the radii.



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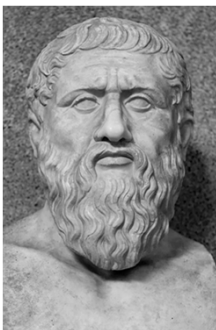
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Plato (427-347 BCE)

Founded "The Academy" in 387 BC which flourished until 529 AD.

Developed a theory of Forms, in his book *Phaedo*, which considers mathematical objects as perfect forms (such as a line having length but no breadth)

Emphasized idea of *proof* and insisted on accurate definitions and clear hypotheses, paving the way for Euclid



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Theaetetus of Athens (417-369 BCE)

Student of Plato's

Creator of solid geometry

First to study octahedron and icosahedron, and construct all five *Platonic solids*



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Eudoxus of Cnidus (408-355 BCE)

Developed a precursor to algebra by developing a theory of proportion

Early work on integration using his *method of exhaustion* by which he determined the area of circles and the volumes of pyramids and cones.




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Menaechmus (380-320 BCE)

Pupil of Eudoxus
 Discovered the conic sections
 First to show that ellipses, parabolas, and hyperbolas are obtained by cutting a cone in a plane not parallel to the base.



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Euclid of Alexandria (325-265 BCE)

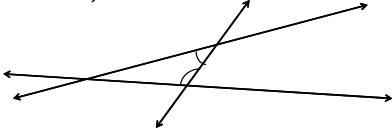


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Euclid's Axioms

Let the following be postulated

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles. (Euclid's Parallel Postulate)



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Euclid's Common Notions

1. Things that are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

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Archimedes of Syracuse (287-212 BCE)

A lot of work on geometry but stayed away from questions on Euclid V



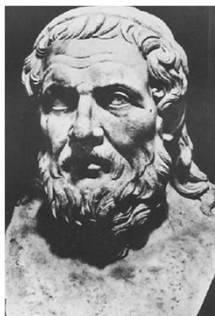
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Apollonius of Perga (262-190 BC)

The *Great Geometer*
 Famous work was *Conics* consisting of 8 Books
Tangencies - showed how to construct the circle which is tangent to three objects
 Computed an approximation for π better than Archimedes.



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Hipparchus of Rhodes (190-120 BC)

Systematically used and documented foundations of trigonometry
 Published several books of trigonometric tables and the methods for calculating them
 Based tables on dividing a circle into 360 degrees with each degree divided into 60 minutes
 Applied trigonometry to astronomy



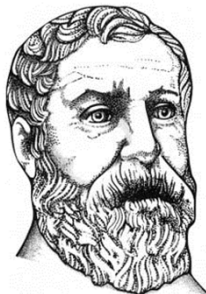
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Heron of Alexandria (10-75 AD)

Wrote *Metrica* which gives methods for computing areas and volumes
 Book I considers areas of plane figures and surfaces of 3D objects, and contains his formula for area of a triangle
 Book II considers volumes of three-dimensional solids.



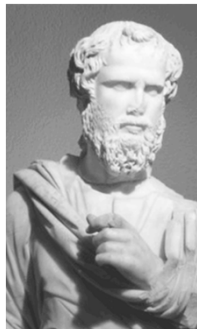
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Menelaus of Alexandria (70-130 AD)

Developed spherical geometry
 Applied spherical geometry to astronomy; Dealt with spherical trigonometry



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Claudius Ptolemy (85-165 AD)

Almagest - Latin form of shortened title "*al mijisti*" of Arabic title "*al-kitabu-l-mijisti*", meaning "The Great Book".

Gave math for geocentric theory of planetary motion

One of the great masterpieces of early mathematical and scientific works



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Pappus of Alexandria (290-350 AD)

Last of great Greek geometers.

Major work in geometry is *Synagoge* - a handbook on a wide variety of topics: arithmetic, mean proportionals, geometrical paradoxes, regular polyhedra, the spiral and quadratrix, trisection, honeycombs, semiregular solids, minimal surfaces, astronomy, and mechanics



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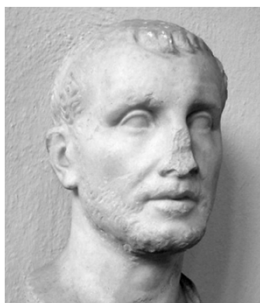
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Posidonius (1st century BC)

Proposed to replace definition of parallel lines by defining them as coplanar lines that are everywhere equidistant from one another

Without Euclid V you cannot prove such lines exist

Also true that statement that parallel lines are equidistant from one another is equivalent to Euclid V.



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Euclid's Axioms - Modern Version

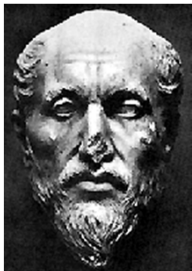
Ptolemy followed with a proof that used the following assumption:
For every line l and every point P not on l , there exists at most one line m through P such that l is parallel to m .

This is equivalent to Euclid V!!

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Proclus (410-485 AD)

Used a limiting process.
 Retained all of Euclid's definitions, all of his assumptions except Euclid V.
 Plan (1) prove on this basis that a line which meets one of two parallels also meets the other, and
 (2) to deduce Euclid V from this proposition.
 Did step (2) correctly.
 No to step (1)



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Other Attempts



Nasiraddin
(1201-1274)



John Wallis (1616-1703)



Adrien Legendre
(1752-1833)



Farkas Bolyai



Girolamo Saccheri (1667-1733)



Johann Heinrich Lambert (1728-1777)

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Euclid V - Equivalent Statements

- *Through a point not on a given line there passes not more than one parallel to the line.*
- *Two lines that are parallel to the same line are parallel to each other.*
- *A line that meets one of two parallels also meets the other.*
- *If two parallels are cut by a transversal, the alternate interior angles are equal.*
- *There exists a triangle whose angle-sum is two right angles.*
- *Parallel lines are equidistant from one another.*
- *There exist two parallel lines whose distance apart never exceeds some finite value.*
- *Similar triangles exist which are not congruent.*
- *Through any three non-collinear points there passes a circle.*
- *Through any point within any angle a line can be drawn which meets both sides of the angle.*
- *There exists a quadrilateral whose angle-sum is four right angles.*
- *Any two parallel lines have a common perpendicular.*

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Overturing Euclidean Geometry



Karl Friedrich Gauss



János Bolyai



Nicolai Ivanovich
Lobachevsky.

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