


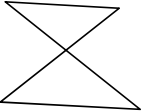
Parallelograms

MA 341 - Topics in Geometry
Lecture 05




Definitions

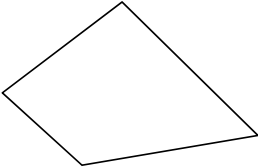
A quadrilateral is a polygon with 4 distinct sides and four vertices. Is there a more precise definition?



P_1



P_2



P_3

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Definitions

Quadrilateral

Quadrangle

Tetragon

These will all mean the same object: 4 line segments that intersect only at the four endpoints (vertices).

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**Important Types of
Quadrilaterals**

Rectangles
Squares
Parallelograms
Rhombus
Trapezoid (trapezium)
Kites

The definitions are important.

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Rectangles

A quadrilateral is a rectangle if ...

it contains 4 right angles.

What is the difference between the definition of an object and properties that an object has?

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Squares

A quadrilateral is a square if ...

it is a rectangle with all sides congruent.

it is a rectangle with adjacent sides congruent.

Are these the same thing?

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Parallelograms

A quadrilateral is a parallelogram if ...

opposite sides are parallel.

Is a rectangle a parallelogram?

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Rhombi or Rhombuses

A quadrilateral is a rhombus if ...

all four sides have the same length.

Is a square a rhombus?

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Trapezoid

A quadrilateral is a trapezoid if ...

contains one pair of parallel sides.

Is a parallelogram a trapezoid?

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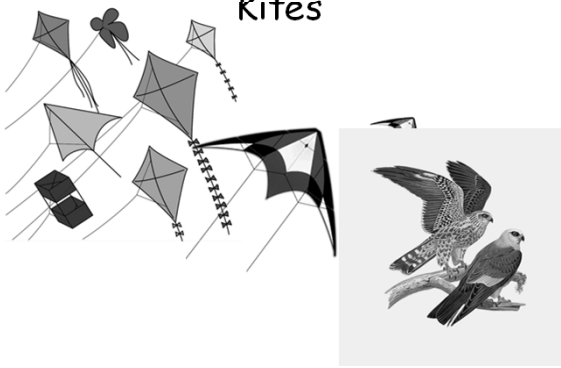
Kites

A quadrilateral is a kite if ...

the four sides can be grouped into two pairs of equal-length sides that are next to each other.

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Kites



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Theorems

Theorem: Opposite sides of a parallelogram are congruent.

Proof

Is the converse to this statement true?

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Theorems

Theorem: In quadrilateral $\square ABCD$ if $AB \cong CD$ and $AD \cong BC$, then $ABCD$ is a parallelogram.

Proof: Homework Problem #1

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Theorems

Theorem: A quadrilateral is a parallelogram if and only if its diagonals bisect one another.

Proof:

(1) If $ABCD$ is a parallelogram then AC and BD bisect each other.

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Theorems

Proof:

(2) In $ABCD$ if AC and BD bisect each other then $ABCD$ is a parallelogram.

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Properties of Parallelograms

- Opposite sides of a parallelogram are equal in length.
- Opposite angles of a parallelogram are equal in measure.
- The area of a parallelogram is the product of the base the height.
- Opposite sides of a parallelogram will never intersect.
- The area is twice the area of a triangle created by one of its diagonals.
- The area equals to the magnitude of the vector cross product of two adjacent sides.

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Properties of Parallelograms

- The diagonals of a parallelogram bisect each other.
- Any non-degenerate affine transformation takes a parallelogram to another parallelogram.
- A parallelogram has rotational symmetry of order 2 (through 180°). If it also has two lines of reflectional symmetry then it must be a rhombus or a rectangle.
- The perimeter of a parallelogram is $2(a + b)$ where a and b are the lengths of adjacent sides.

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Properties of Parallelograms

- Consecutive angles of a parallelogram are supplementary.

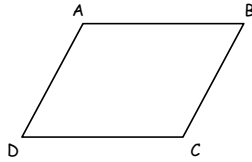
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Interesting Facts - Parallelograms

- Start with a parallelogram, ABCD.



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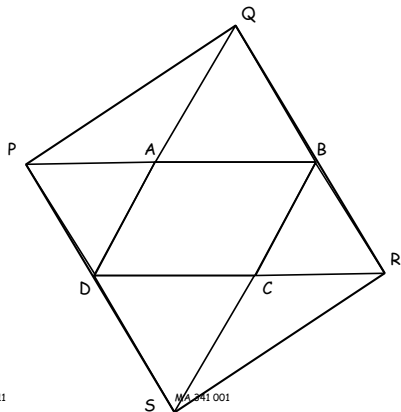
Interesting Facts - Parallelograms

- Construct an equilateral triangle on the outside of each side of the parallelogram.
- The vertices, PQRS, form a parallelogram.

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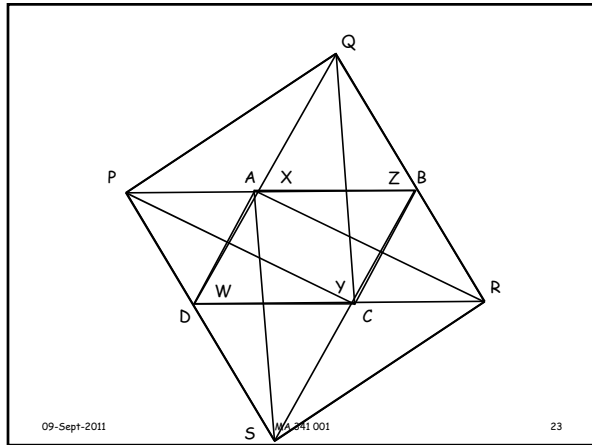
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Interesting Facts - Parallelograms

- Construct an equilateral triangle on the inside of each side of the parallelogram PQRS.

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Interesting Facts - Parallelograms

- XZYW is a parallelogram.
- Is XZYW = ABCD or is it coincidence?

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Interesting Facts - Parallelograms

- We can write P in terms of A and D .
- In fact, we can write the third vertex of an equilateral triangle in terms of the other two vertices - using complex numbers!

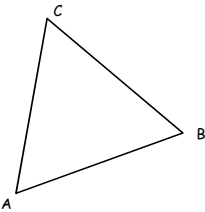
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Aside-Equilateral Triangles

In the plane of $\triangle ABC$ we can choose a Cartesian coordinate system so that we can identify points in the plane with complex numbers - $(a,b) \leftrightarrow a + bi$.

Let T denote the rotation through 120° in the positive (counterclockwise) direction. This means that $T^3 = 1$, or, since $T \neq 1$,

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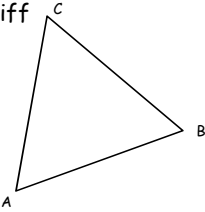
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Aside-Equilateral Triangles

$T^3 - 1 = 0$
 $(T - 1)(T^2 + T + 1) = 0$ and since $T \neq 1$,
 $T^2 + T + 1 = 0$

Theorem: $\triangle ABC$ is equilateral iff
 $A + TB + T^2C = 0$.

Theorem: $\triangle ABC$ is equilateral iff
 $A + TB + T^2C = 0$.



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Aside-Equilateral Triangles

Proof:
 ΔABC is equilateral iff each side could be obtained from another side by a rotation through 60° around their common vertex.

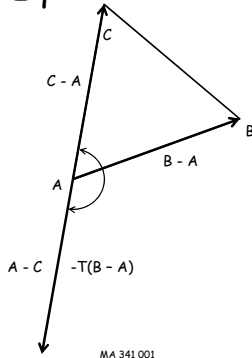
A rotation through 120° in the opposite direction, brings one side in the direction exactly opposite to the other making their sum equal to 0.

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Aside-Equilateral Triangles



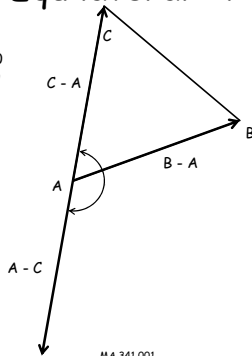
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Aside-Equilateral Triangles

$$\begin{aligned} T(B - A) + C - A &= 0 \\ C + TB - (1+T)A &= 0 \\ C &= (1+T)A - TB \end{aligned}$$



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$P = (1+T)D - TA$
 $Q = (1+T)A - TB$
 $R = (1+T)B - TC$
 $S = (1+T)C - TD$

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$T^2 + T + 1 = 0$
 $X = (1+T)S - TR$
 $= (1+T)((1+T)C - TD) - T((1+T)B - TC)$
 $= ((1+T)^2 + T^2)C - (1+T)TD - T(1+T)B$
 $= (-1 + 2 + 2T + 2T^2)C - (T^2 + T)D - (T + T)B$
 $= -C + D + B$

BUT $A - B = D - C$

So

$X = A - B + B = A$

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Interesting Facts - Parallelograms

- Start with a parallelogram, ABCD.

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• Draw the squares on the outside of the sides of the parallelogram.

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• The segments joining the centers of these squares is a square.

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Varignon's Theorem

• Let $ABCD$ be a convex quadrilateral and $X, Y, Z,$ and W the midpoints of the sides. $XYZW$ is a parallelogram.

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