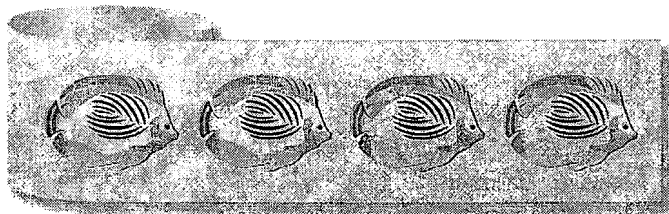
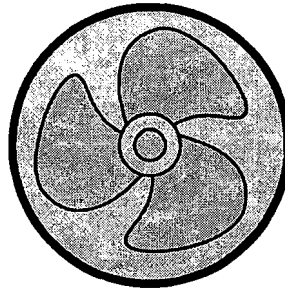
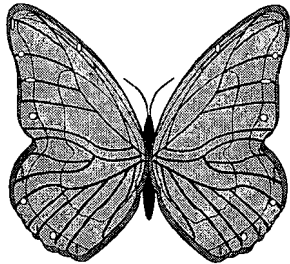


Connected Math:  
Kaleidoscopes,  
Hub caps,  
and Mirrors

## Three Types of Symmetry

**W**hen part of a design is repeated to make a balanced pattern, we say the design has **symmetry**. Artists use symmetry to make designs that are pleasing to the eye. Architects use symmetry to produce a sense of balance in their buildings. Symmetry is also a feature of animals, plants, and mechanical objects.

The butterfly, fan, and ribbon below illustrate three kinds of symmetry.

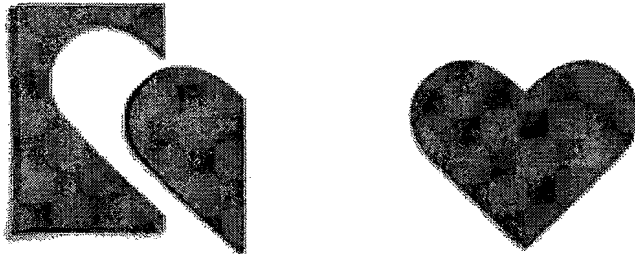


### Getting Ready for Problem 1.1

- What part of each design is repeated to make a balanced pattern that allows us to say the three figures have symmetry?
- How do the figures suggest different kinds of symmetry?

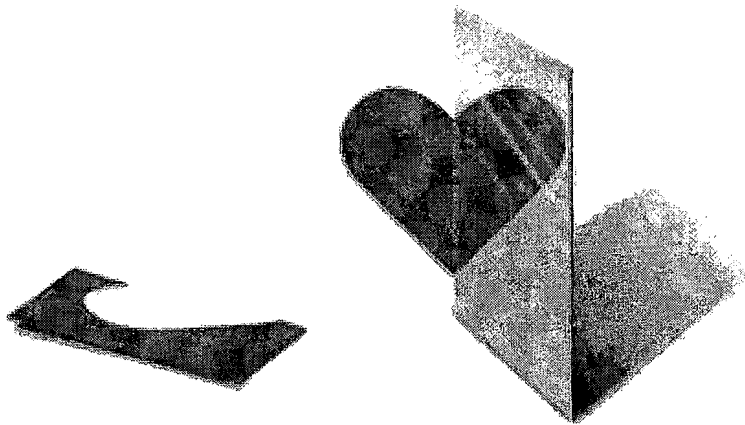
## 1.1 Reflection Symmetry

You have probably made simple heart shapes by folding and cutting paper as shown below.



The resulting heart shape has **reflection symmetry**, which is sometimes called *mirror symmetry* or *line symmetry*. The fold shows the **line of symmetry**. A line of symmetry divides a figure into halves that are mirror images.

If you place a mirror on a line of symmetry, you will see half of the figure reflected in the mirror. The combination of the half-figure and its reflection will have the same size and shape as the original figure. You can use a mirror to check a design for symmetry and to locate the line of symmetry.

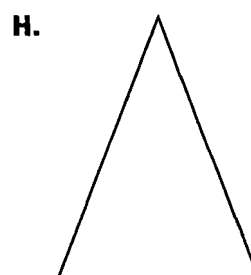
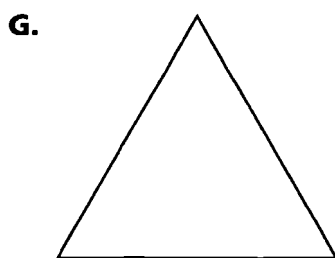
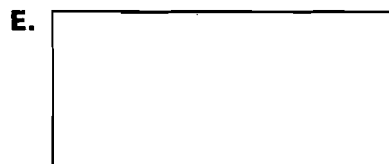
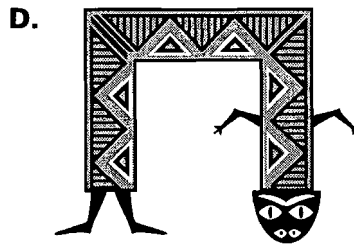
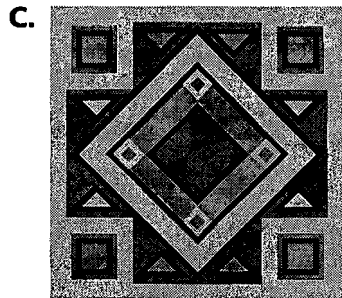
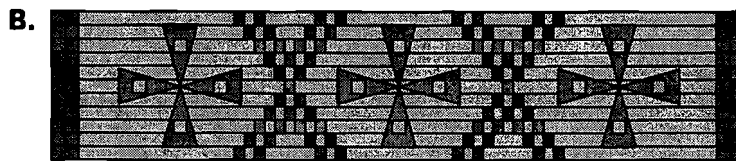
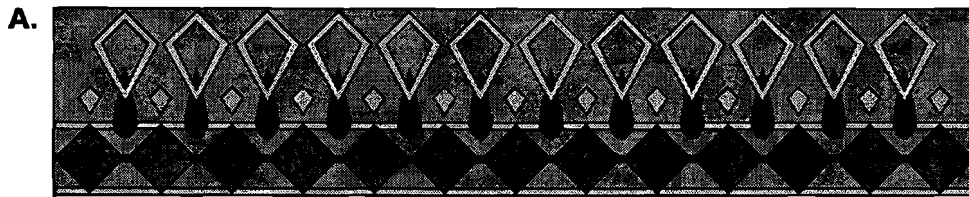


You can also use tracing paper to check for reflection symmetry. Trace the figure and the possible line of symmetry. Then reflect the tracing over the possible line of symmetry. If the reflected tracing fits exactly on the original figure, the figure has reflection symmetry.

*What happens to the line of symmetry when you reflect the tracing and match it with the original figure? Does its location change?*

## Problem 1-1 Reflection Symmetry

Use a mirror, tracing paper, or other tools to find all lines of symmetry in each design or figure.

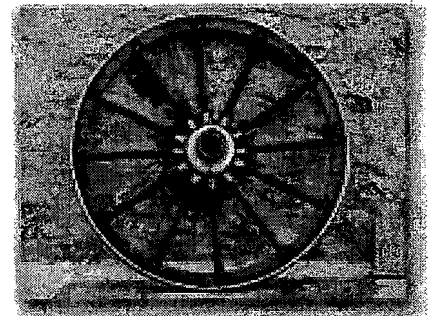
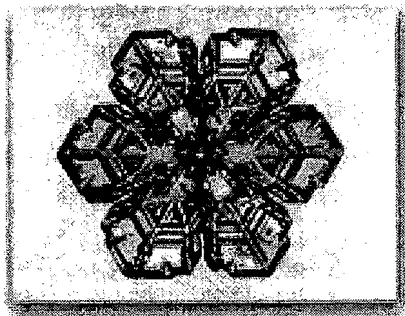
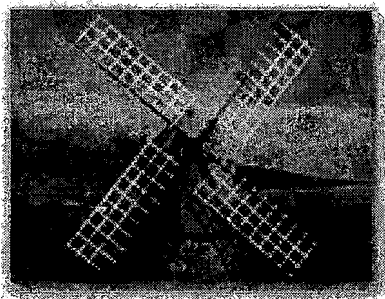
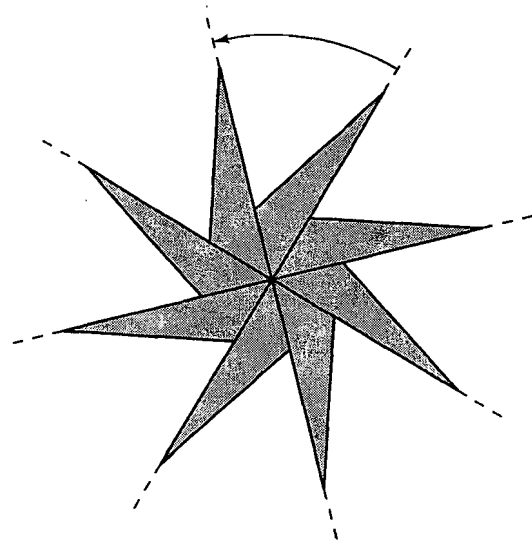


**CE** Homework starts on page 15.

## 1.2 Rotation Symmetry

The pinwheel design at the right does not have reflection symmetry. However, it can be turned less than a full turn around its center point in a counterclockwise direction to positions in which it looks the same as it does in its original position. Figures with this property are said to have **rotation symmetry**.

The windmill, snowflake, and wagon wheel pictured below also have rotation symmetry.



Which two of the three objects pictured above also have reflection symmetry?

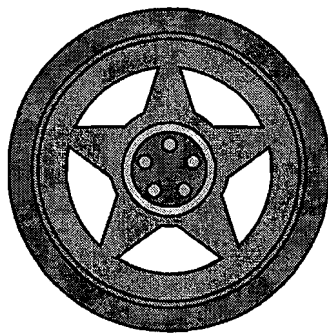
To describe the rotation symmetry in a figure, you need to specify two things:

- The *center of rotation*. This is the fixed point about which you rotate the figure.
- The *angle of rotation*. This is the *smallest* angle through which you can turn the figure in a counterclockwise direction so that it looks the same as it does in its original position.

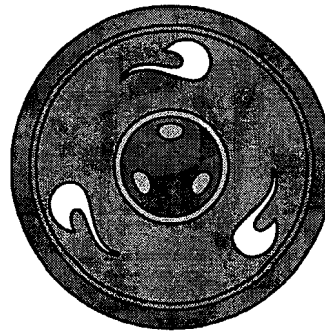
There are several rotation angles that move the pinwheel design above to a position where it looks like the original. In this problem, you will consider how these angles are related to the angle of rotation.

## Problem 1.2 Rotation Symmetry

- A.** List all the turns of less than  $360^\circ$  that will rotate the pinwheel design to a position in which it looks the same as what is pictured. What is the angle of rotation for the pinwheel design?
- B.** In parts (1)–(3), list all the turns of less than  $360^\circ$  that will rotate the object to a position in which it looks the same as what is pictured. Then give the angle of rotation.
1. the windmill      2. the snowflake      3. the wagon wheel
- C.** Look at your answers for Questions A and B. For each object or figure, tell how the listed angles are related to the angle of rotation.
- D.** The hubcaps below have rotation symmetry. Complete parts (1) and (2) for each hubcap.



Hubcap 1



Hubcap 2

1. On a copy of the hubcap, mark the center of rotation. Then, find all the turns of less than  $360^\circ$  that will rotate the hubcap to a position in which it looks the same as what is pictured.
2. Tell whether the hubcap has reflection symmetry. If it does, draw all the lines of symmetry.
- E.** Draw a hubcap design that has rotation symmetry with a  $90^\circ$  angle of rotation but no reflection symmetry.
- F.** Draw a hubcap design that has rotation symmetry with a  $60^\circ$  angle of rotation and at least one line of symmetry.
- G.** Investigate whether rectangles and parallelograms have rotation symmetry. Make sketches. For the shape(s) with rotation symmetry, give the center and angle of rotation.

*active math*  
*online*

For: Hubcap Maker  
Visit: PHSchool.com  
Web Code: apd-5102

**CE** Homework starts on page 15.

# 1.3

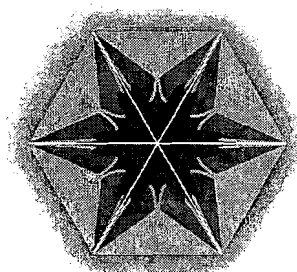
## Symmetry in Kaleidoscope Designs

A *kaleidoscope* (kuh ly duh skohp) is a tube containing colored beads or pieces of glass and carefully placed mirrors. When you hold a kaleidoscope up to your eye and turn the tube, you see colorful symmetric patterns.

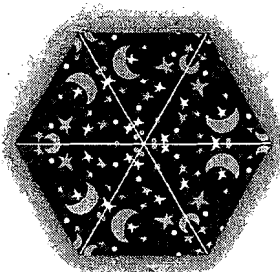
The kaleidoscope was patented in 1817 by the Scottish scientist Sir David Brewster. Brewster was intrigued by the science of nature. He developed kaleidoscopes to simulate the designs he saw in the world around him.



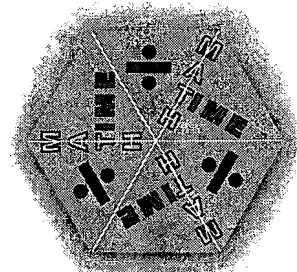
Five of the designs below are called *kaleidoscope designs* because they are similar to designs you would see if you looked through a kaleidoscope.



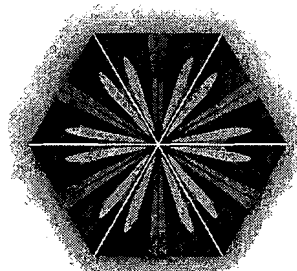
A



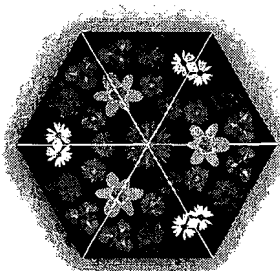
B



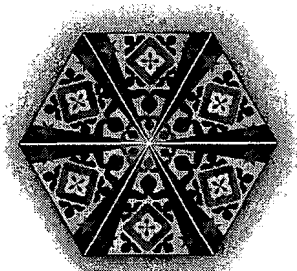
C



D



E

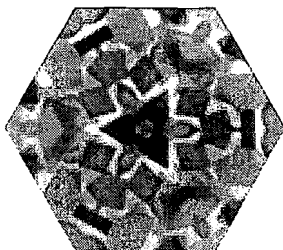


F

### Problem 1.3 Analyzing Symmetries

Use what you know about reflection and rotation symmetry to analyze the six designs.

- A. Locate all the lines of symmetry in the designs.
- B. Give the angles of rotation for the designs with rotation symmetry.
- C.
  1. Make a table showing the number of lines of symmetry and the angle of rotation for each design.
  2. What relationship, if any, do you see between the number of lines of symmetry and the angle of rotation?
  3. Analyze the kaleidoscope design below to see whether it confirms your relationship.

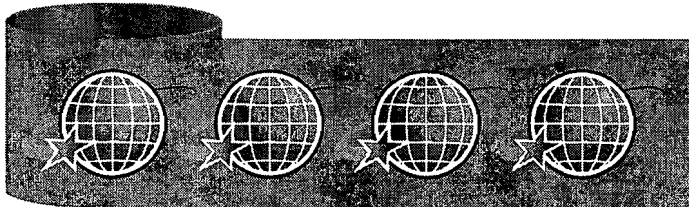
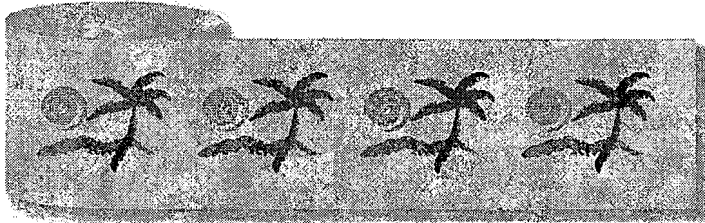


- D. Each of the designs can be made by repeating a small piece of the design. We call this piece the **basic design element**. For each design, sketch or outline the basic design element.
- E. One of the designs is *not* a kaleidoscope design. That is, it is not similar to a design you would see if you looked through a kaleidoscope. Which design do you think it is? Why?

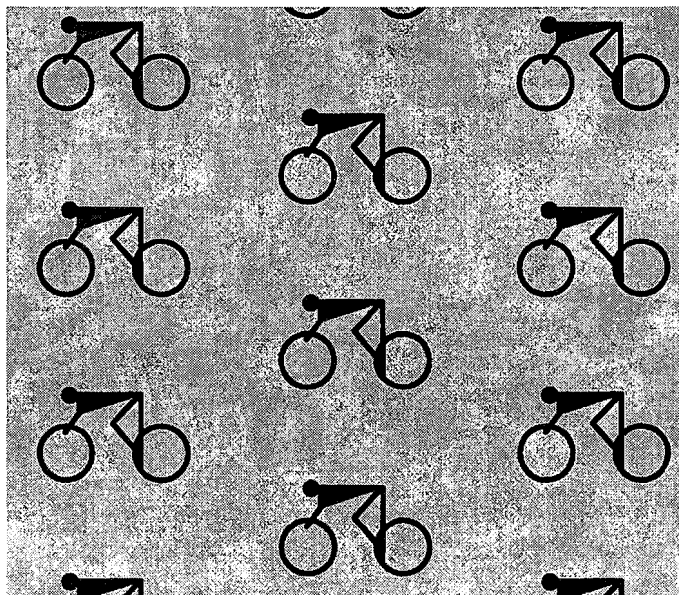
**CE** Homework starts on page 15.

## 1.4 Translation Symmetry

The next three designs are examples of “strip patterns.” You can draw a strip pattern by repeating a basic design element at regular intervals to the left and right of the original.



You can use a similar design strategy to make a “wallpaper pattern” like the one below.



Making a strip pattern or a wallpaper design requires a series of “draw and move” steps. You draw a basic design element. Then, you slide your pencil to a new position and repeat the element. You slide in the same way to a new position and repeat the element again, and so on. The slide movements from one position to the next are called **translations**.

### Getting Ready for Problem 1.4

- Suppose the strip patterns on the previous page extend forever in both directions. Describe how you can move each infinite pattern so it looks exactly the same as it does in its original position.
- Suppose the wallpaper pattern on the previous page extends forever in all directions. Describe how you can move the infinite pattern so it looks exactly the same as it does in its original position.

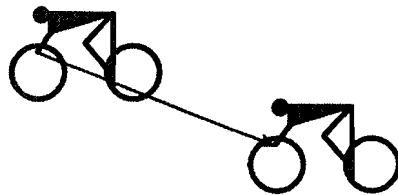


A design has **translation symmetry** if you can slide the whole design to a position in which it looks exactly the same as it did in its original position.

To describe translation symmetry, you need to specify the distance and direction of the translation. You can do this by drawing an arrow indicating the slide that would move the design “onto itself.”

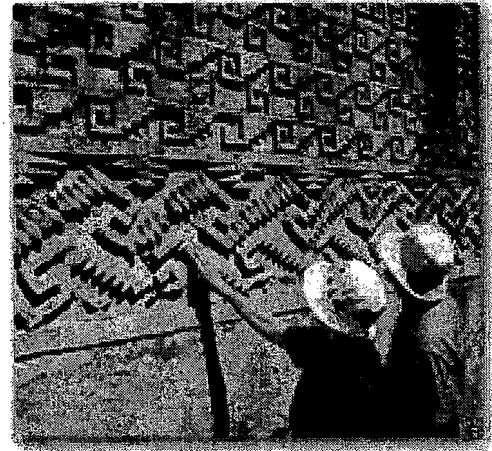
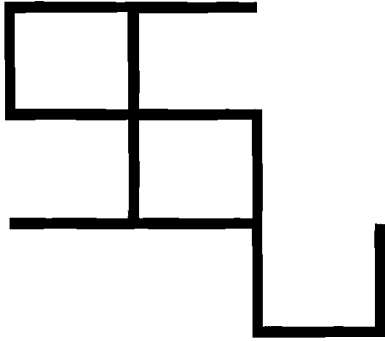
Questions about translation symmetry are of two kinds.

- Given a basic design element, how can you use draw-and-slide operations to produce a pattern with translation symmetry?
- How can you tell whether a given design has translation symmetry?

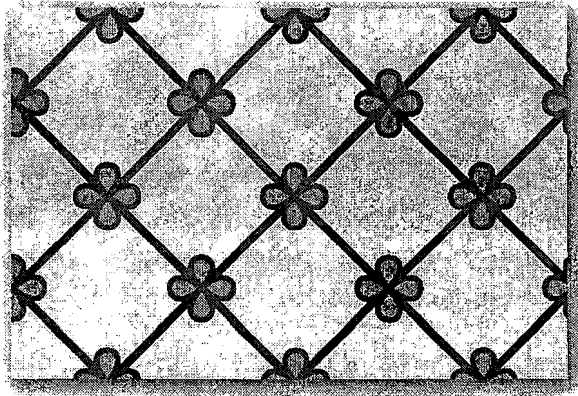


## Problem 1.4 Translation Symmetry

- A. Cut a long strip of paper about one inch wide. Use the basic design element below to draw a strip pattern on the paper. The resulting strip pattern can be found in fabrics made by the Mayan people who live in Central America.



- B. 1. Below is a part of a design that extends forever in all directions. Outline a basic design element that can be used to make the entire design using only translations.



2. Describe precisely how the basic design element can be copied and translated to produce the pattern. Your description should include diagrams with arrows and measures of distances.

**ACE** Homework starts on page 15.

## Applications

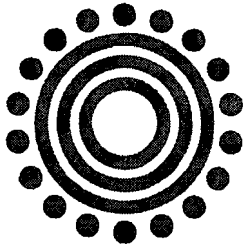
1. Which capital letters have reflection symmetry? For each one that does, describe all the lines of symmetry.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

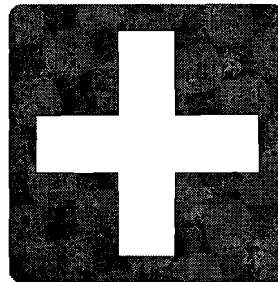
Tell whether the design has reflection symmetry. If it does, sketch the design and draw all the lines of symmetry.

**Go Online**  
 PHSchool.com  
 For: Multiple-Choice Skills  
 Practice  
 Web Code: apa-5154

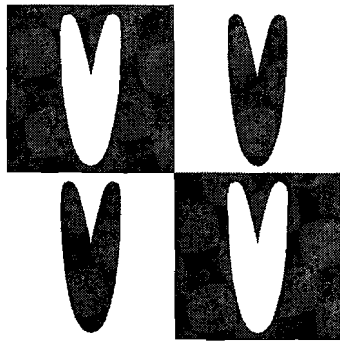
2.



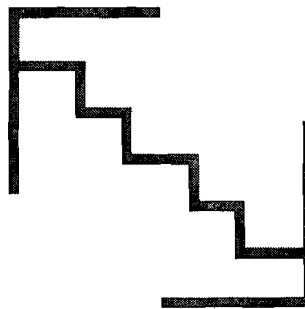
3.



4.

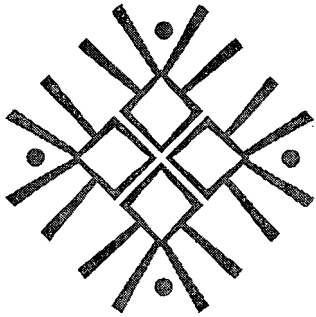


5.

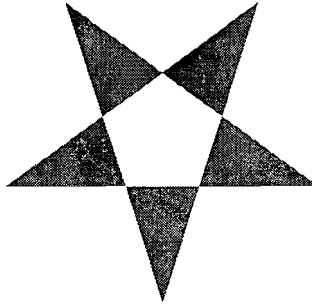


For Exercises 6–9, tell whether the design has reflection symmetry. If it does, sketch the design and draw all the lines of symmetry.

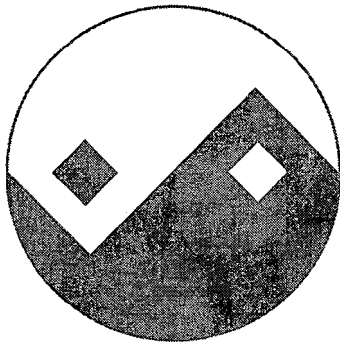
6.



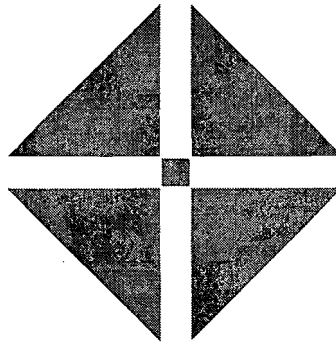
7.



8.



9.



10. Which of the figures in Exercises 2–9 have rotation symmetry? For each one that does, find the angle of rotation and tell which multiples of this angle rotate the figure to a position in which it looks like the original.

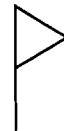
**Homework Help Online**  
PHSchool.com

For: Help with Exercise 10  
Web Code: ape-5110

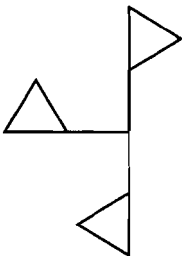
11. Which capital letters have rotation symmetry? For each one that does, give the angle of rotation.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

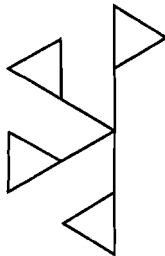
For Exercises 12–14, use the flag shape at the right as a basic design element. Complete a design with rotation symmetry and give the angle of rotation.



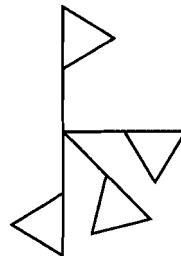
12.



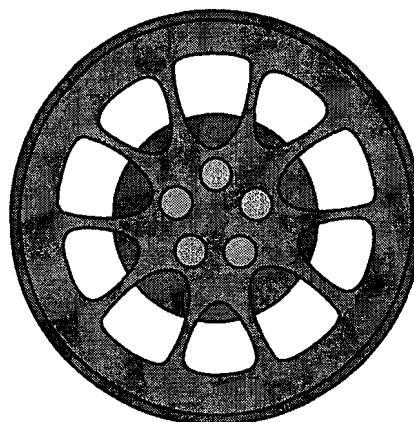
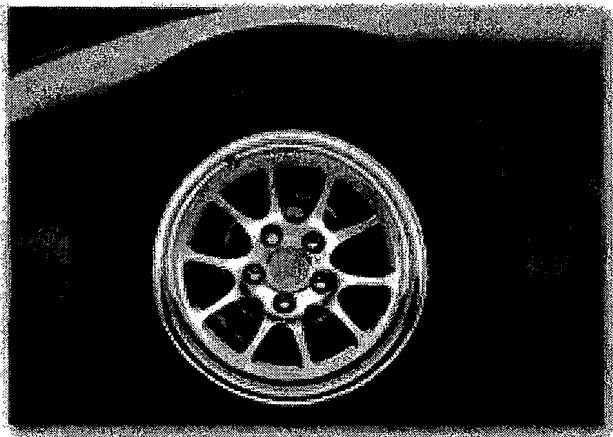
13.



14.

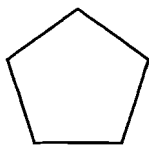


15. a. Give the angle of rotation for the hubcap at the right.  
b. Copy the hubcap and draw the lines of symmetry.

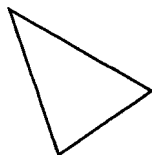


16. Draw a hubcap design that has rotation symmetry with a  $120^\circ$  angle of rotation and at least one line of symmetry.
17. **Multiple Choice** Which figure does not have rotation symmetry?

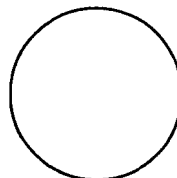
A.



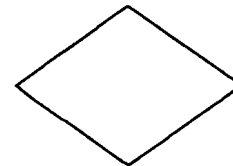
B.



C.

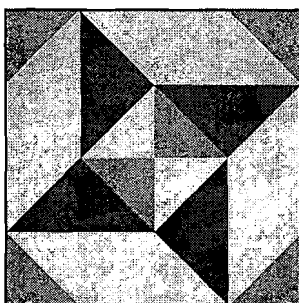


D.

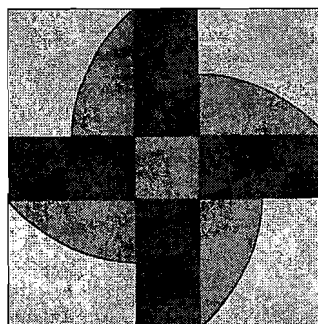


Describe the reflection and rotation symmetries for each traditional quilt design.

18.

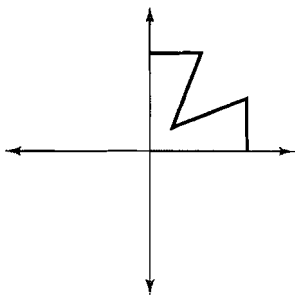


19.

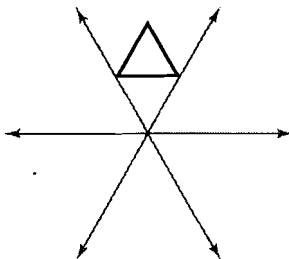


For Exercises 20–23, copy the drawing. Then, draw a design with the given lines as lines of symmetry.

20.



21.



22.

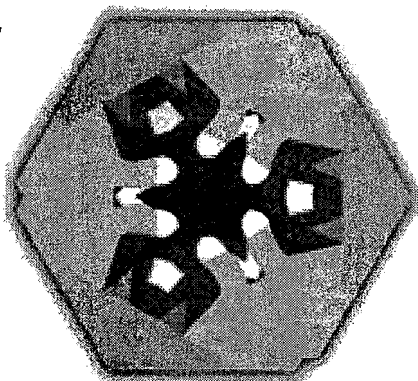


23.

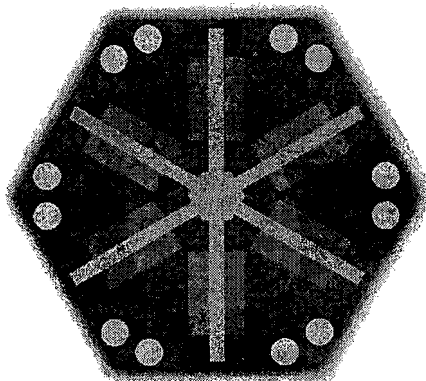


For Exercises 24 and 25, describe the rotation and reflection symmetries for the design.

24.



25.



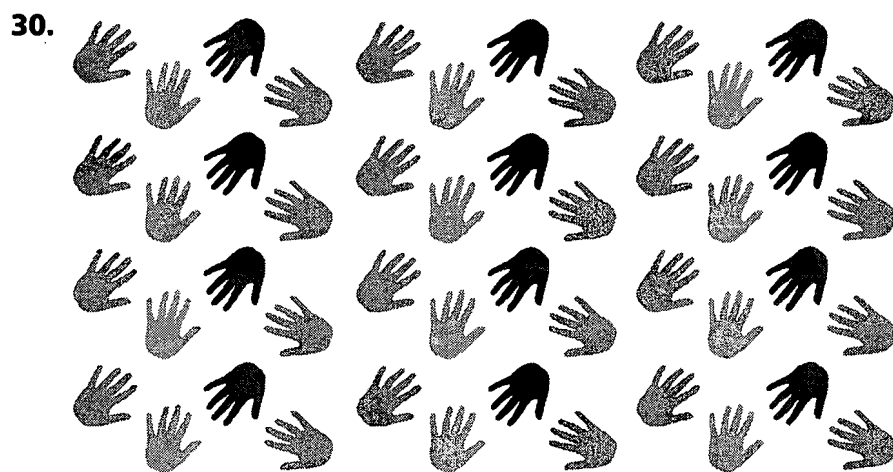
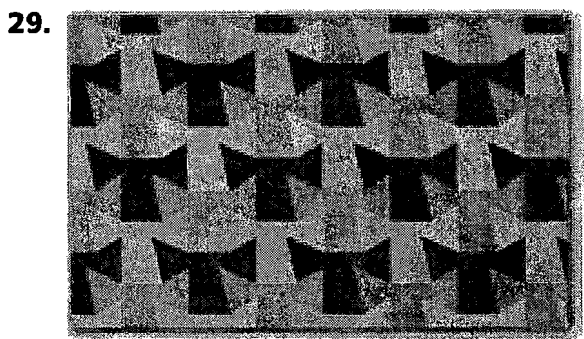
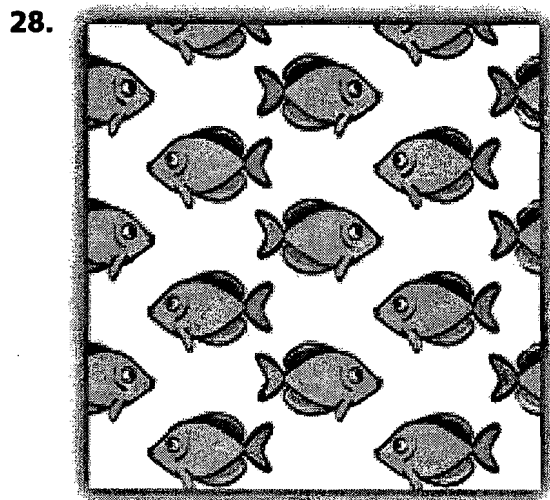
26. In parts (a) and (b), use a capital letter as the basic design element.

- a. Sketch a strip pattern with reflection symmetry only.
- b. Sketch a strip pattern with reflection symmetry and rotation symmetry.

27. a. Using the capital letter X as the basic design element, sketch part of a wallpaper pattern.

- b. Show, with arrows, all the translations that slide the design onto itself.
- c. Draw the lines of symmetry for your design.
- d. Describe the rotation symmetries in your design.

For Exercises 28–30, identify the basic design element for the wallpaper design. Then, describe how this basic design element can be copied and translated to produce the pattern. Include diagrams with arrows and measures of distance.



## Connections

Consider state names written in capital letters (for example, ALABAMA or MICHIGAN).

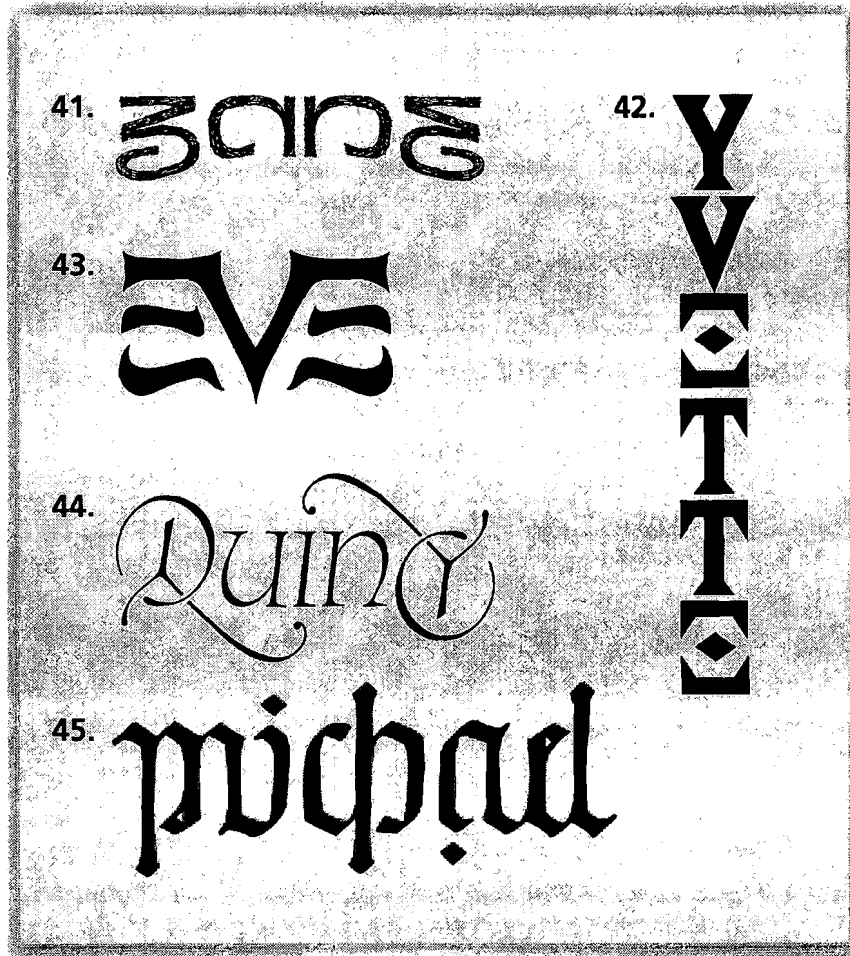
- 31. Find a state name that has reflection symmetry when written horizontally.
- 32. Find a state name that has reflection symmetry when written vertically.

33. Write a word or phrase that has
- reflection symmetry when written horizontally
  - reflection symmetry when written vertically

Draw an example of each type of polygon. Draw all the lines of symmetry. If the polygon has rotation symmetry, identify the center and angle of rotation.

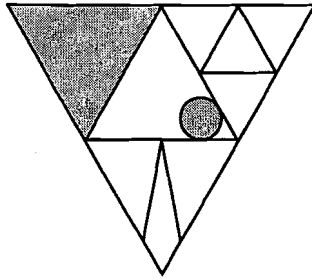
- |                                   |                          |
|-----------------------------------|--------------------------|
| 34. square                        | 35. non-square rectangle |
| 36. non-rectangular parallelogram | 37. isosceles triangle   |
| 38. equilateral triangle          | 39. non-square rhombus   |
| 40. isosceles trapezoid           |                          |

The designs in Exercises 41–45 are actually first names. Describe the symmetries in each name. Then, write the name in standard lettering.

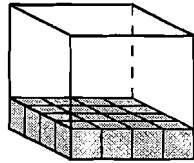


SOURCE: ScottKim.com

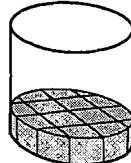
46. Copy the design below. Then use tracing paper to help you sketch a full kaleidoscope design from this basic design element.



47. Prisms and cylinders each have two congruent faces one of which is the base. The drawings show a prism and a cylinder that are 4 centimeters high. Each is filled with a layer of centimeter cubes. The cubes in the bottom layer of the cylinder include some parts of cubes to make an exact covering of the base of the cylinder.



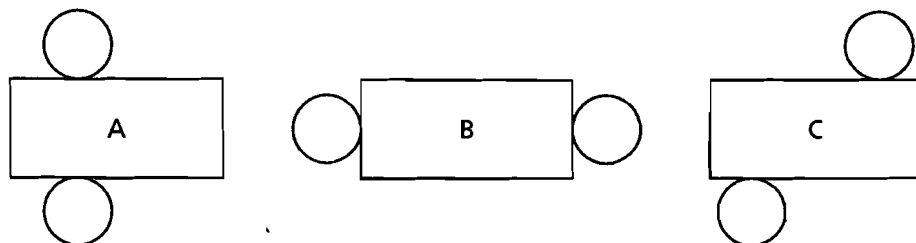
Rectangular Prism



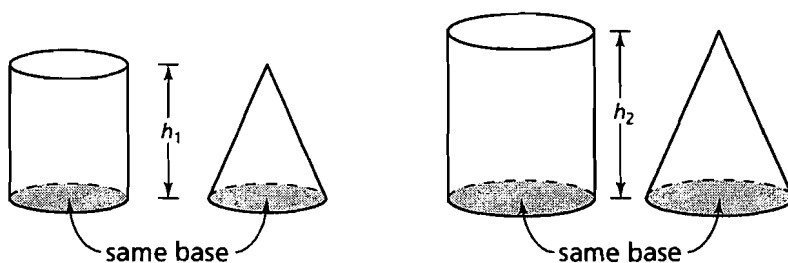
Cylinder

- How many layers will it take to fill the prism?
- What is the volume of the prism?
- What formula does this suggest for finding the volume of a prism?
- How many layers will it take to fill the cylinder?
- What is the volume of the cylinder?
- What formula does this suggest for finding the volume of a cylinder?
- How do the volumes of the prism and cylinder that are given compare?

48. At the right is a cylinder and below are three patterns that show the faces of that cylinder.

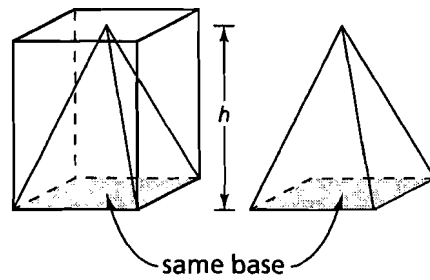


- Which pattern is not a flat pattern for a cylinder?
  - Which patterns have reflection symmetry? Make sketches to show the lines of symmetry.
  - Which patterns have rotation symmetry? Make sketches to show the centers of rotation.
  - What measurements would you need to make to find the volume of a cylinder?
  - How do your measurements connect to the formula for finding the volume?
  - What is the volume of the cylinder if the base has a radius of 6 cm and the height is 15 cm?
49. Tyrone has been experimenting with cylinders and cones. He has made several paper cones and cylinders with the same height and base. He fills the cone with colored sand and pours it into the cylinder with the same base and height. To his surprise it always takes 3 cones of sand to exactly fill the cylinder.



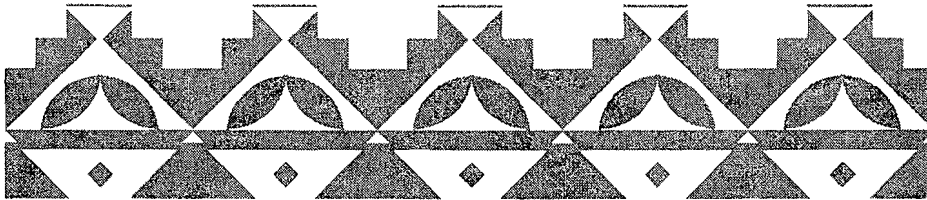
- The height of a cylinder is 8 cm and its base has a radius of 3 cm. What is the volume of the cone with the same base and height?
- What formula for the volume of a cone does Tyrone's experiments suggest?
- Tyrone makes some prisms and pyramids that are related. Each pair has the same base and height. He conducts the same experiment and finds that it always takes 3 pyramids to fill the matching prism exactly. What formula does this suggest for the volume of a pyramid?

- d. If the base of a square pyramid is 7 cm on each edge and its height is 12 cm, what is its volume?

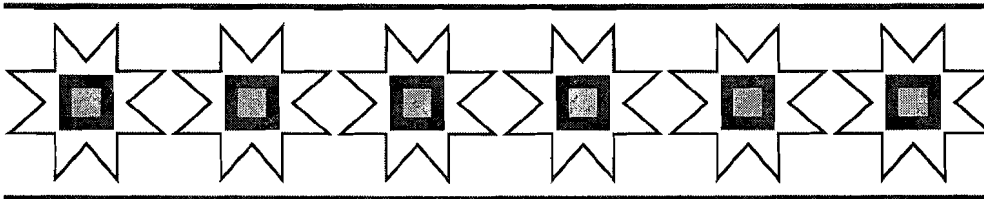


These strip patterns are found in crafts in traditional cultures around the world. For each pattern, identify the basic design element and describe the lines of symmetry and center and angle of rotation. (Assume each design continues without bound to the left and right.)

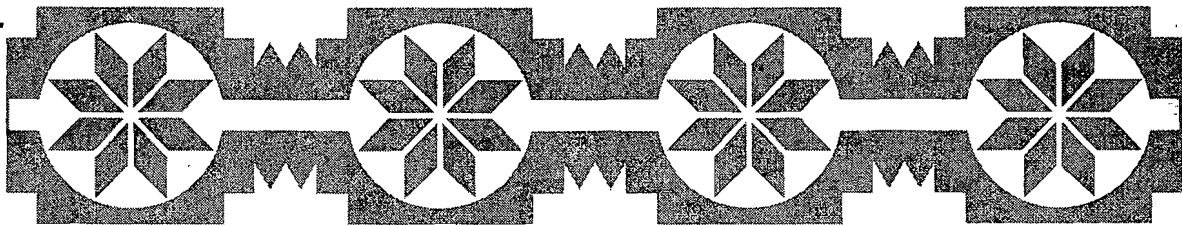
50.



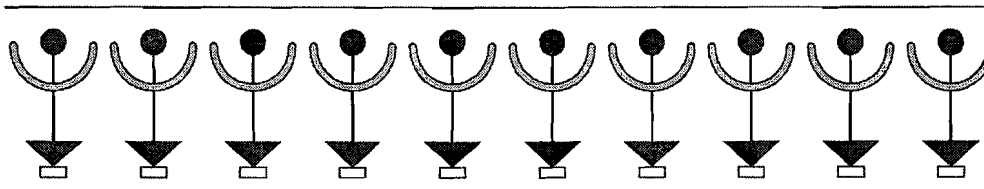
51.



52.

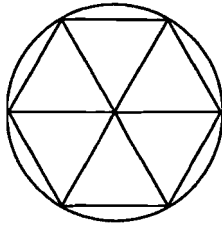


53.

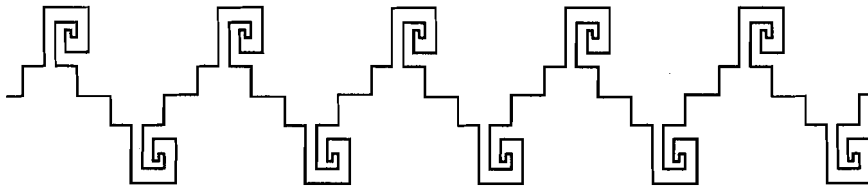


## Extensions

54. A regular hexagon can be enclosed by a circle as shown in the following sketch.

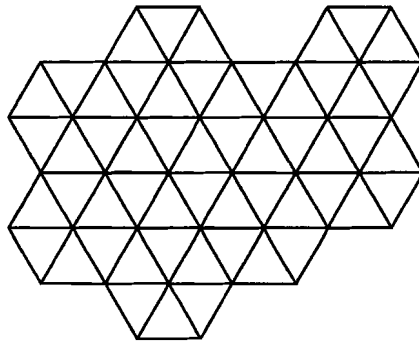


- Describe all the symmetries in the figure.
  - What are the angle measures of the triangles in the figure? Explain how you know.
  - Suppose you started with a circle with the center marked. How can you use the symmetries you observed in part (a) to construct
    - a regular triangle?
    - a regular hexagon?
  - How are your answers to parts (a), (b), and (c) related to the symmetries in any kaleidoscope pattern?
55. In this investigation, you studied designs with reflection, rotation, and translation symmetries. The design below is a bit different from those you have seen.

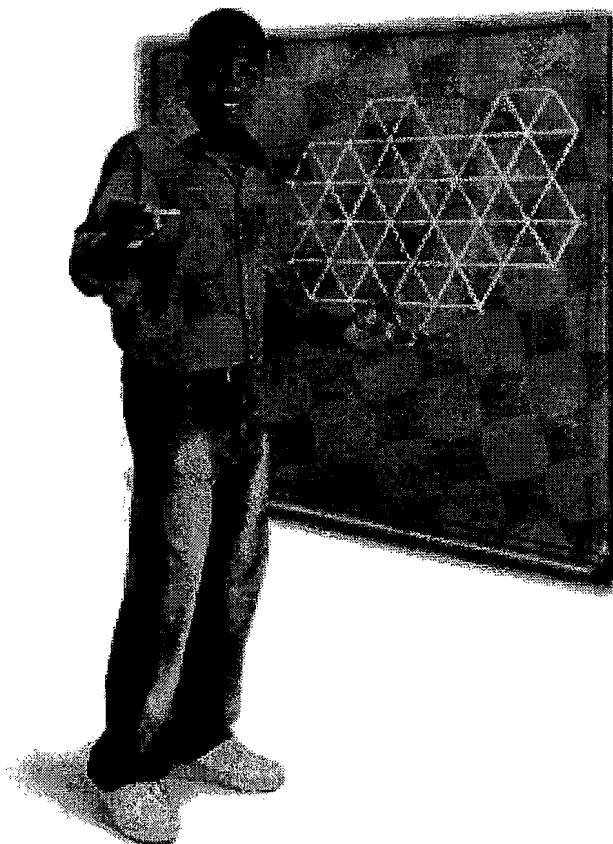


- Trace a basic design element from which the rest of the pattern can be produced using only translations.
- Trace a smaller basic design element from which the rest of the pattern can be produced by using a translation followed by a reflection. Indicate the line of reflection and the length and direction of the translation.
- The movement required in part (b) to generate the pattern is called a *glide reflection*. What is the difference between a reflection and a glide reflection?

56. Using a capital letter as the basic design element, make a strip pattern using only glide reflections.
57. a. Make a sketch of the design below. Outline a basic design element that can be translated to produce the entire design.  
b. Draw arrows to specify the distance and direction(s) you need to slide your basic element to make this design.



- c. Can you translate a single triangle to make the design above? Explain.



# Mathematical Reflections 1

**In this investigation, you identified and made symmetric designs. These questions will help you summarize what you have learned.**

---

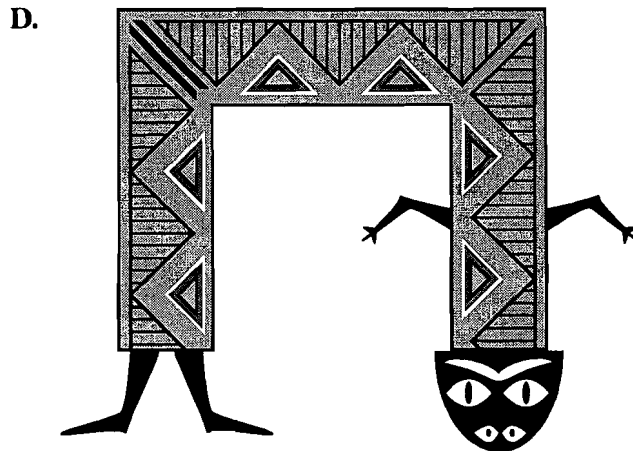
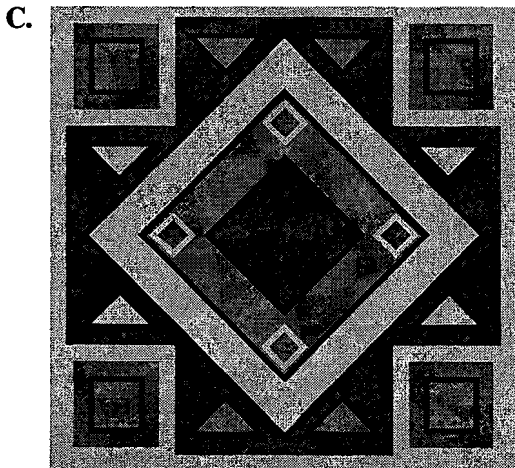
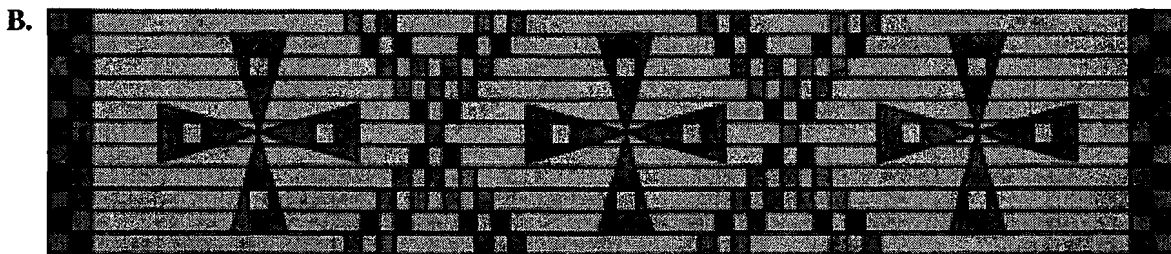
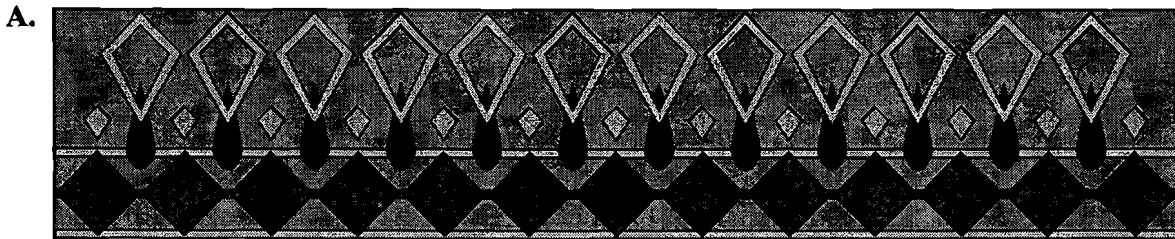
Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then, write a summary of your findings in your notebook.

1. How would you explain to someone what it means for a figure to have
  - a. reflection symmetry?
  - b. rotation symmetry?
  - c. translation symmetry?
2. How can you use various drawing tools to test or draw figures with
  - a. reflection symmetry?
  - b. rotation symmetry?
  - c. translation symmetry?

# Labsheet 1.1A

Kaleidoscopes, Hubcaps, and Mirrors

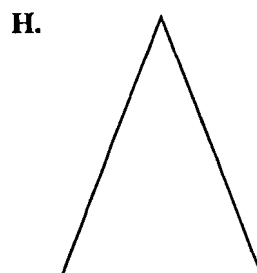
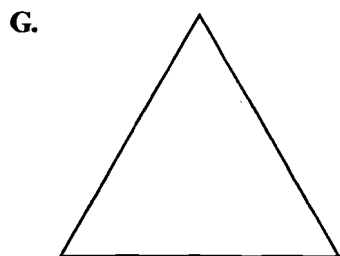
## Questions A–D



# Labsheet 1.1B

.....  
Kaleidoscopes, Hubcaps, and Mirrors

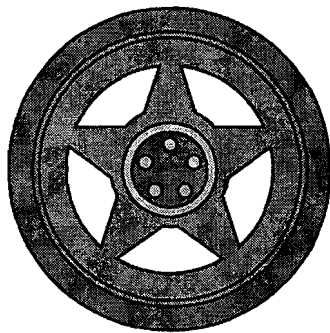
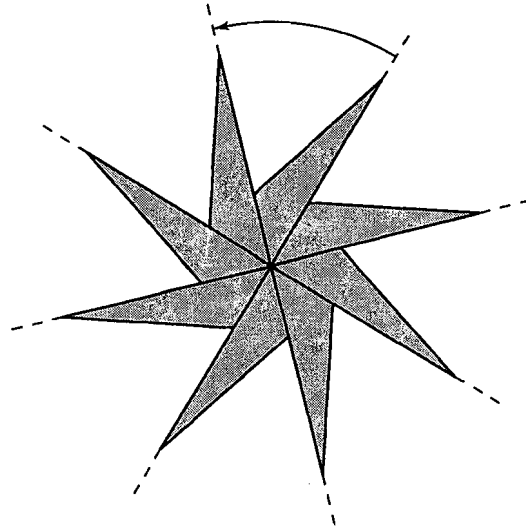
## Questions E-H



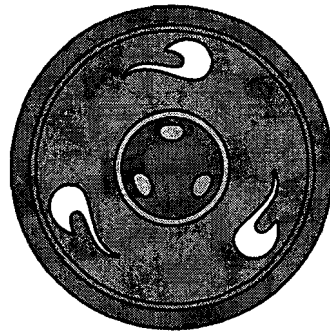
# Labsheet 1.2

Kaleidoscopes, Hubcaps, and Mirrors

## Pinwheel and Hubcaps



Hubcap 1

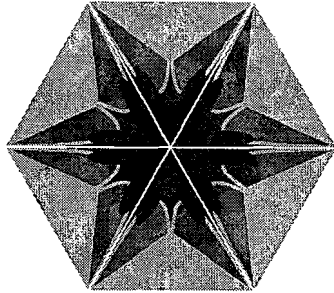


Hubcap 2

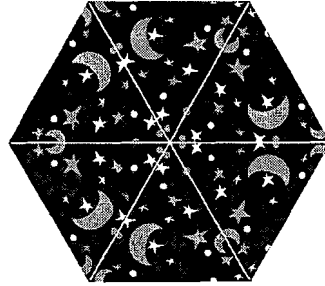
# Labsheet 1.3

Kaleidoscopes, Hubcaps, and Mirrors

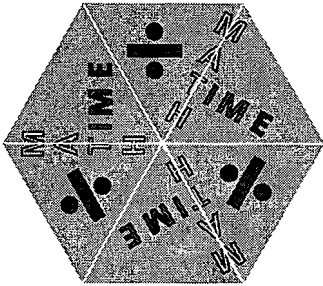
## Kaleidoscope Designs



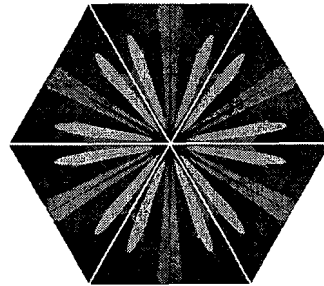
A



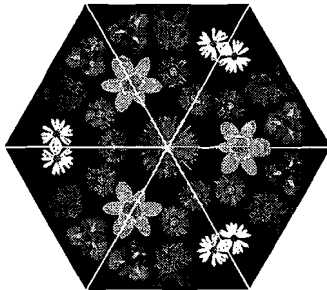
B



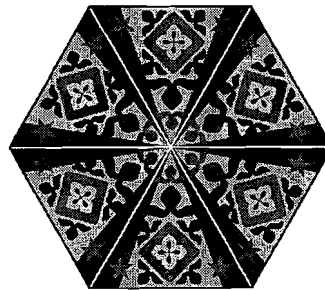
C



D



E

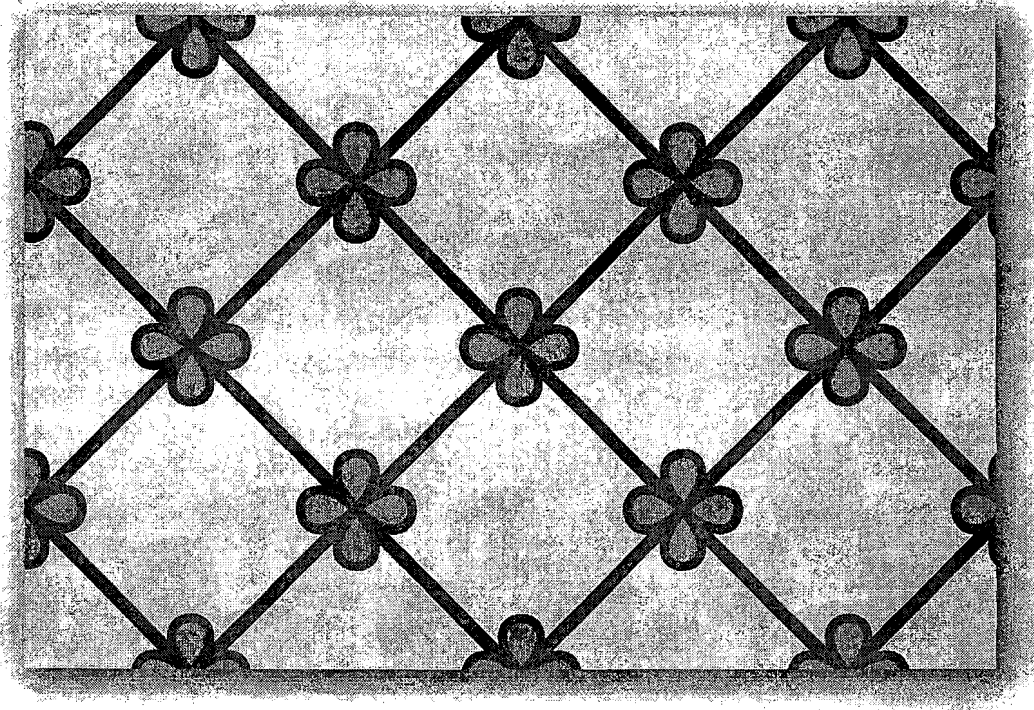


F

# Labsheet 1.4

Kaleidoscopes, Hubcaps, and Mirrors

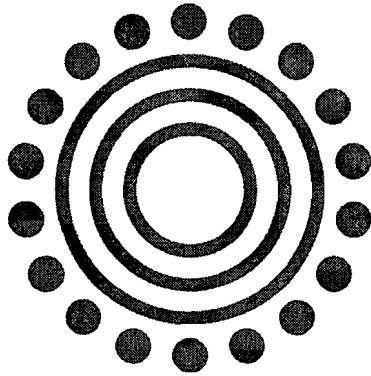
## Question B



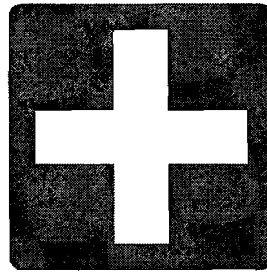
# Labsheet 1ACE Exercises 2-5

Kaleidoscopes, Hubcaps, and Mirrors

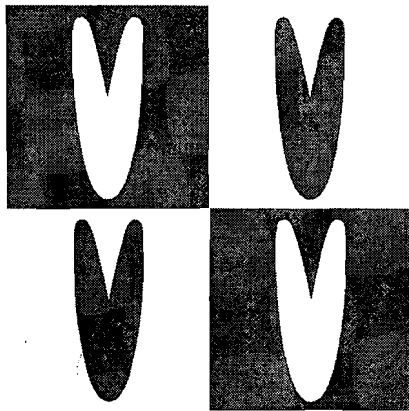
2.



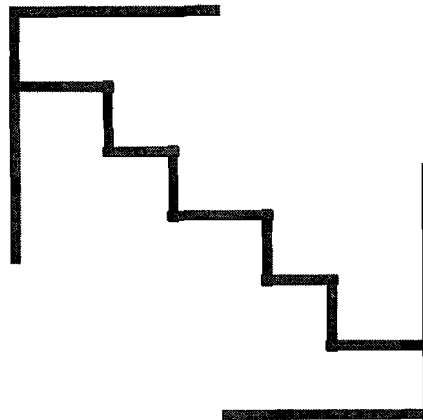
3.



4.



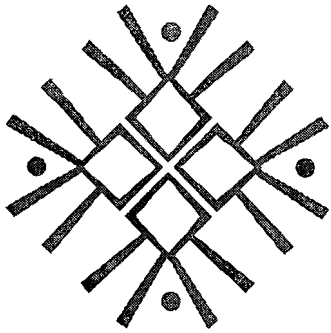
5.



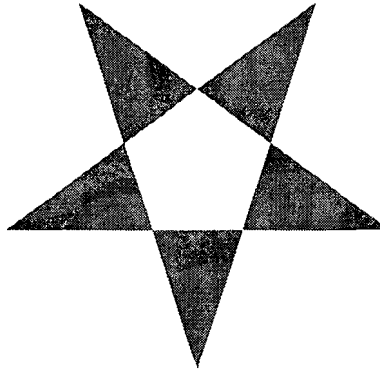
# Labsheet 1ACE Exercises 6-9

.....  
Kaleidoscopes, Hubcaps, and Mirrors

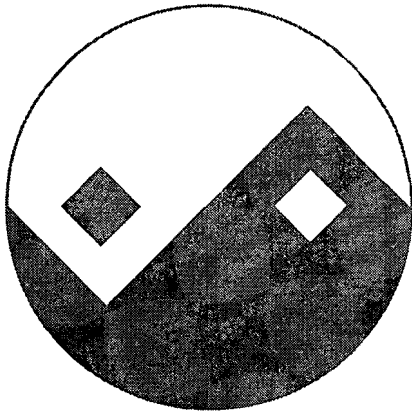
6.



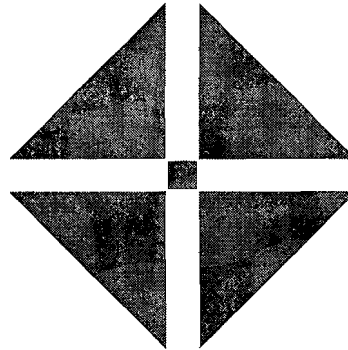
7.



8.



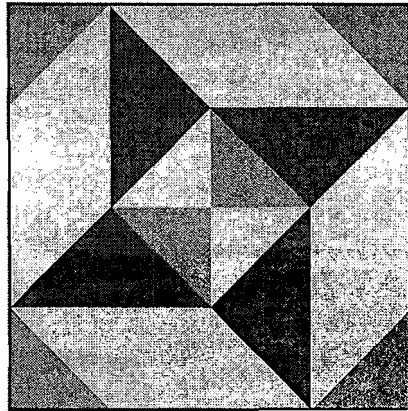
9.



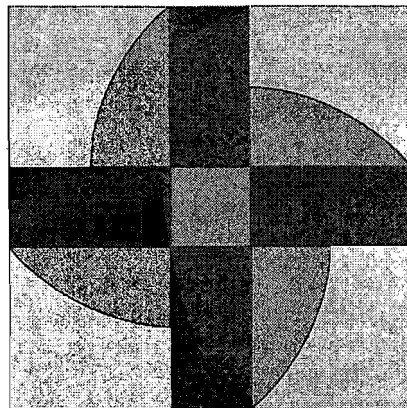
# Labsheet IACE Exercises 18-19

Kaleidoscopes, Hubcaps, and Mirrors

18.



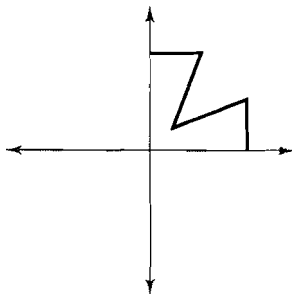
19.



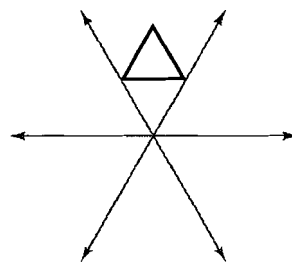
# Labsheet 1ACE Exercises 20-23

.....  
Kaleidoscopes, Hubcaps, and Mirrors

20.



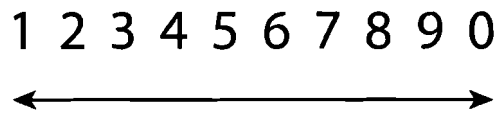
21.



22.



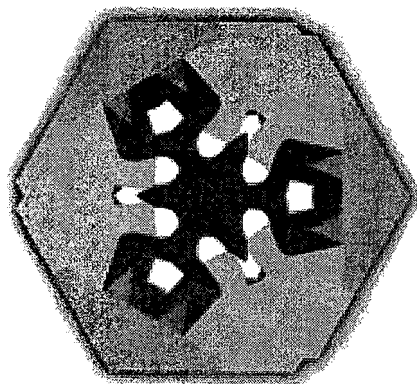
23.



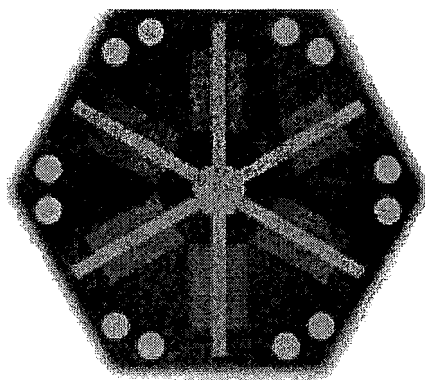
# Labsheet 1ACE Exercises 24–25

.....  
Kaleidoscopes, Hubcaps, and Mirrors

24.



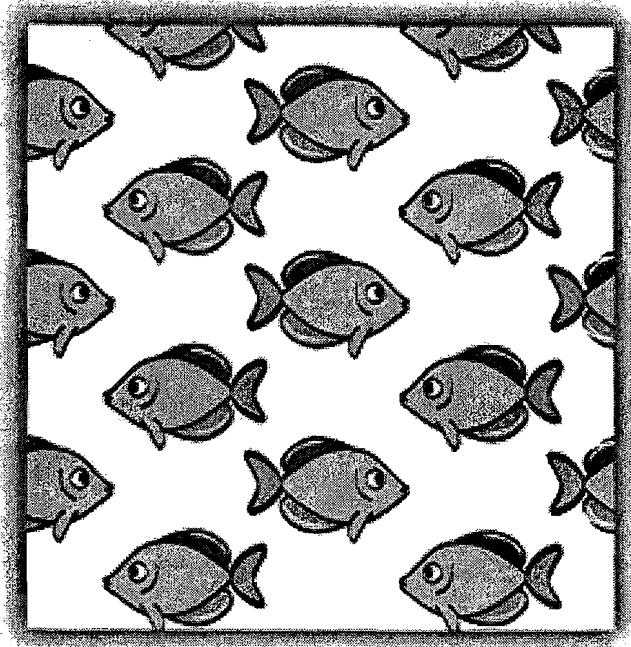
25.



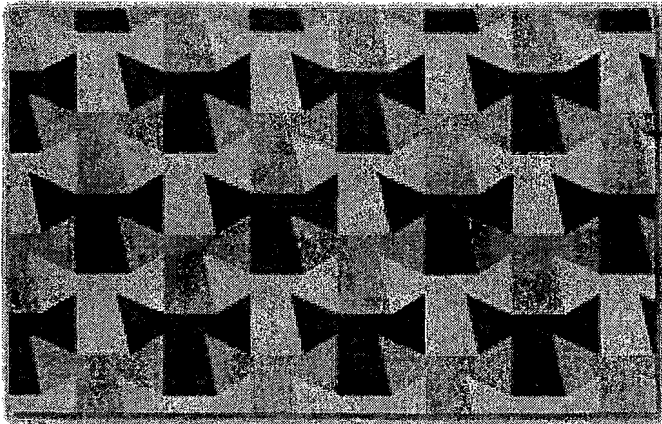
# Labsheet IACE Exercises 28-29

Kaleidoscopes, Hubcaps, and Mirrors

28.



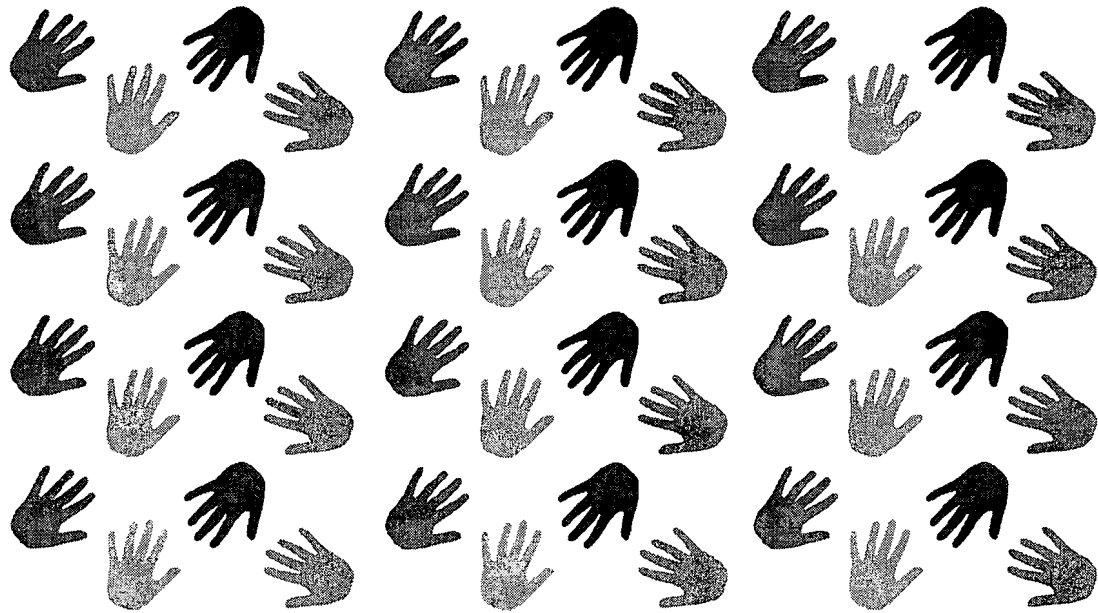
29.



# Labsheet 1ACE Exercise 30

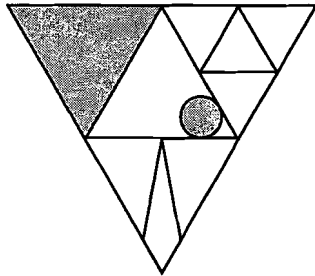
Kaleidoscopes, Hubcaps, and Mirrors

30.



# Labsheet IACE Exercise 46

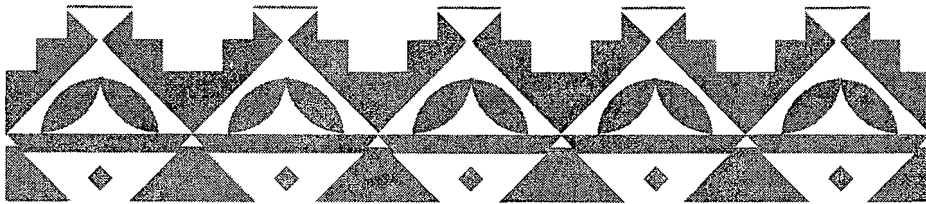
.....  
Kaleidoscopes, Hubcaps, and Mirrors



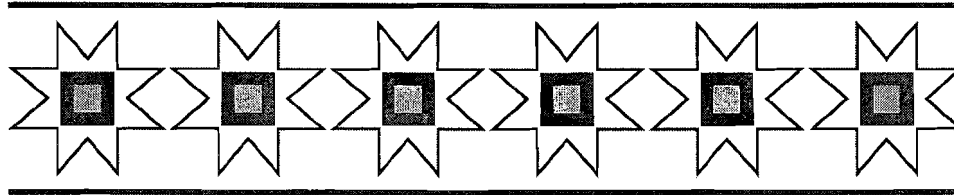
# Labsheet 1ACE Exercises 50-53

Kaleidoscopes, Hubcaps, and Mirrors

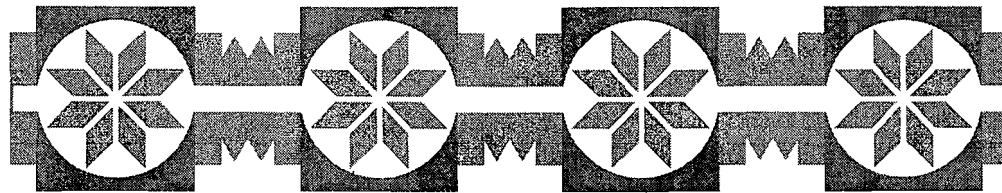
50.



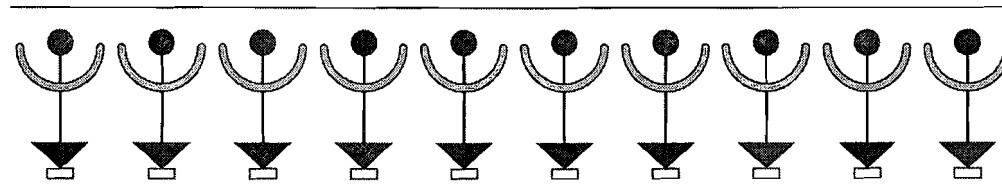
51.



52.



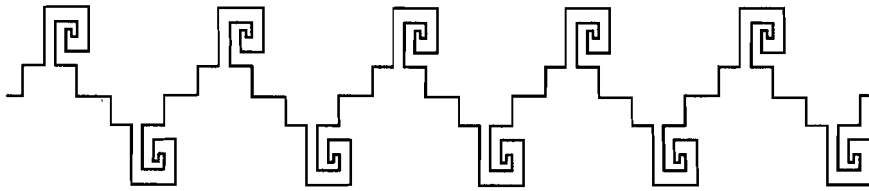
53.



# Labsheet 1ACE Exercises 55 and 57

Kaleidoscopes, Hubcaps, and Mirrors

55.



57.

