Investigation 3

Connected Math: Shapes and Designs

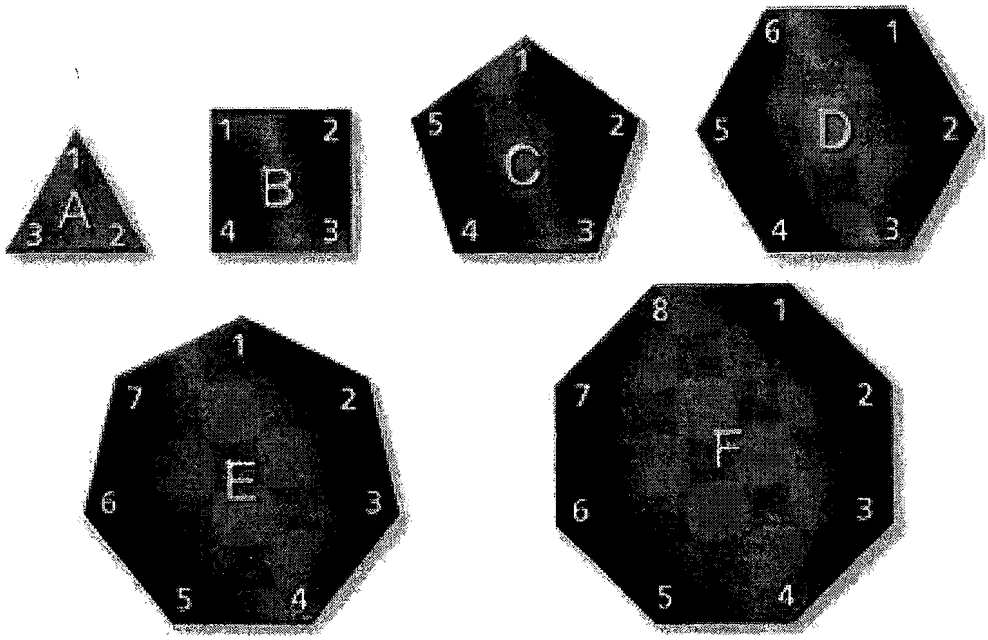
Polygon Properties and Tiling

You learned about angles and angle measure in Investigations 1 and 2. What you learned can help you figure out some useful properties of the angles of a polygon. Let's start with the sum of the measures of all the inside angles at the vertices of a polygon. This sum is called the **angle sum** of a polygon.

3.1 Angle Sums of Regular Polygons

TEKS / TAKS **6(6)B** Identify relationships involving angles in triangles and quadrilaterals. **6(8)C** Measure angles. **6(13)A** Make conjectures.

Below are six regular polygons that are already familiar to you.



What is the angle sum of each figure?

Do you see a pattern relating the number of sides to the angle sum?

Problem 3.1 Angle Sums of Regular Polygons

A. 1. In Problem 2.3, you measured the angles of some regular polygons—triangles, squares, and hexagons. Record the number of sides, the angle measures, and the angle sum of a triangle, square, and hexagon in a table like the one below.

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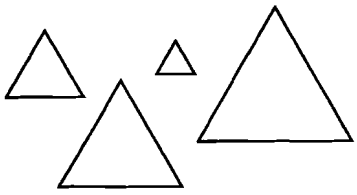
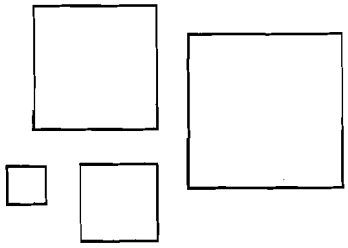
Polygon	Number of Sides	Measure of an Angle	Angle Sum
Triangle			
Square			
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

2. Measure an angle of the regular pentagon and regular octagon from your Shapes Set. Record the measures of the angles and the angle sums in your table. What patterns do you see?

3. Use your patterns to fill in the table for a regular polygon with seven, nine, and ten sides.

B. Below are two sets of regular polygons of different sizes. Do the same patterns relating the number of sides, the measures of the angles, and the angle sums apply to these shapes? Explain.





C. Describe how you could find the angle sum of a regular polygon that has N sides.

ACE Homework starts on page 62.

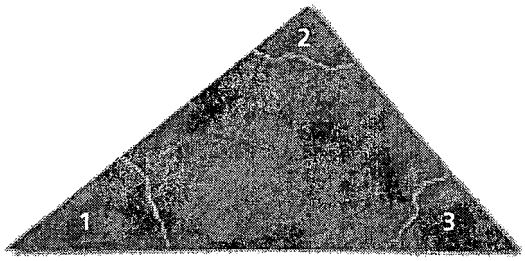
3.2 Angle Sums of Any Polygon

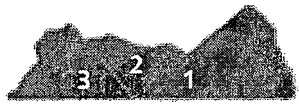
TEKS / TAKS 6(6)B Identify relationships involving angles in triangles and quadrilaterals.
6(13)A Make conjectures from patterns.

Do the patterns that you observed for the angle sum of regular polygons apply to all polygons?

Getting Ready for Problem 3.2

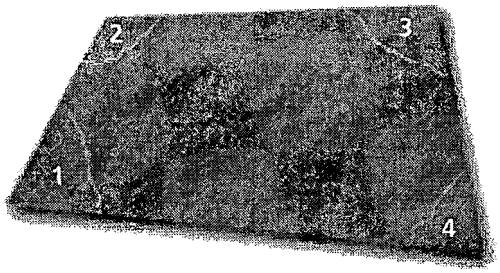
Suppose you tear the three corners off of a triangle. You can arrange them this way:

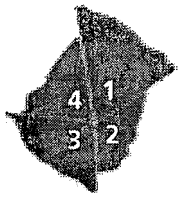




- Based on the picture, what is the sum of angles 1, 2, and 3? How do you know?
- Make a conjecture about the angle sum of any triangle.

You could do the same thing with a quadrilateral.



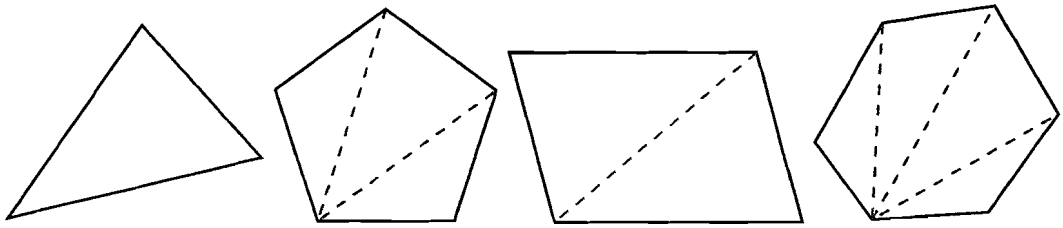


- Based on the picture, what is the sum of angles 1, 2, 3, and 4? How do you know?
- Make a conjecture about the angle sum of any quadrilateral.
- Do similar patterns hold for other polygons?

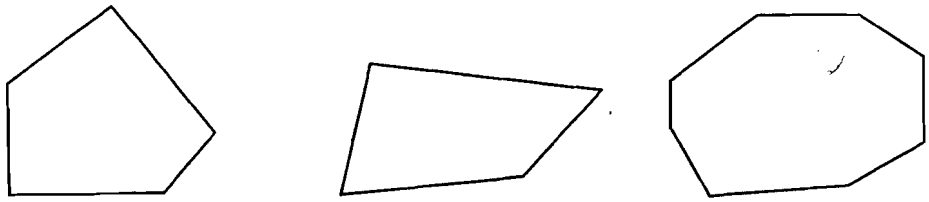
Problem 3.2 Angle Sums of Any Polygon

Tia and Cody claim that the angle sum of any polygon is the same as the angle sum of a regular polygon with the same number of sides. They use diagrams to illustrate their reasoning.

A. Tia divides polygons into triangles by drawing all the *diagonals* of the polygons from one vertex, as in the diagrams below:

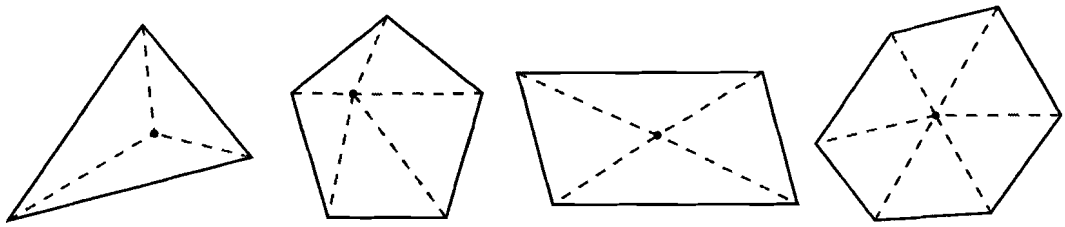


1. Study Tia's drawings. How can you use Tia's method to find the angle sum of each polygon?
2. Copy these three polygons. Use Tia's method to find the angle sum of each polygon.



3. Does Tia's method work for any polygon? Explain.

B. Cody also discovered a method for finding the angle sum of any polygon. He starts by drawing line segments from a point within the polygon to each vertex.



1. Study Cody's drawings. How can you use Cody's method to find the angle sum of each polygon?
2. Copy the three polygons from Question A part (2). Use Cody's method to find the angle sum of each polygon.
3. Does Cody's method work for any polygon? Explain.

C. In Problem 3.1, you found a pattern relating the number of sides of a regular polygon to the angle sum. Does the same pattern hold for any polygon? Explain.

ACE **Homework starts on page 62.**

3.3 Back to the Bees!

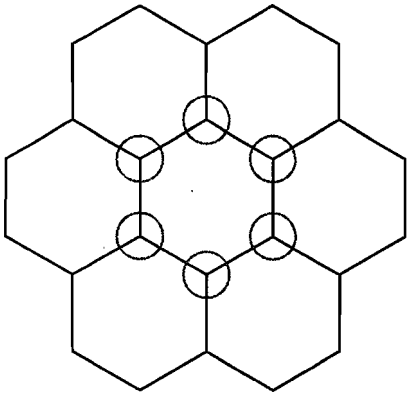
TEKS / TAKS 6(11)D* Select tools such as manipulatives to solve problems.
6(13)B Validate conclusions using mathematical properties.

* not tested on TAKS

When the honeybees make a honeycomb, they build tubes. As the tubes press together, they become hexagonal in shape. So, the surface of a honeycomb looks like it is covered with hexagons. We can't ask honeybees why their honeycomb construction results in hexagons. However, there are some mathematical properties of hexagons that may offer explanations.

Below is a tiling of regular hexagons. Notice that three angles fit together exactly around any point in the tiling.

Why do these regular hexagons fit together so neatly?



In Problem 1.3, you experimented to find which regular polygons could tile a surface.

What are the properties of these shapes that allow them to fit together so neatly around a point?

Problem 3.3 Angles in Tilings

A. In Problem 1.3, you explored tilings made from a single type of regular polygon. You found that only equilateral triangles, squares, and regular hexagons could be used to tile a surface.

1. For each of these shapes, make a tiling and sketch the results.
2. In each case, explain why copies of the shape fit neatly around a point.

B. In Problem 1.3, you also found that regular pentagons, regular heptagons, and regular octagons could not be used to tile a surface. Explain why copies of these polygons do not fit neatly around a point.

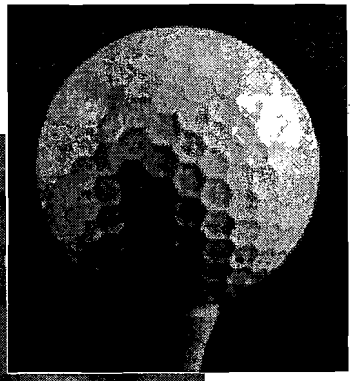
C. 1. Find tilings using combinations of two or more shapes from your Shapes Set. Sketch your results.

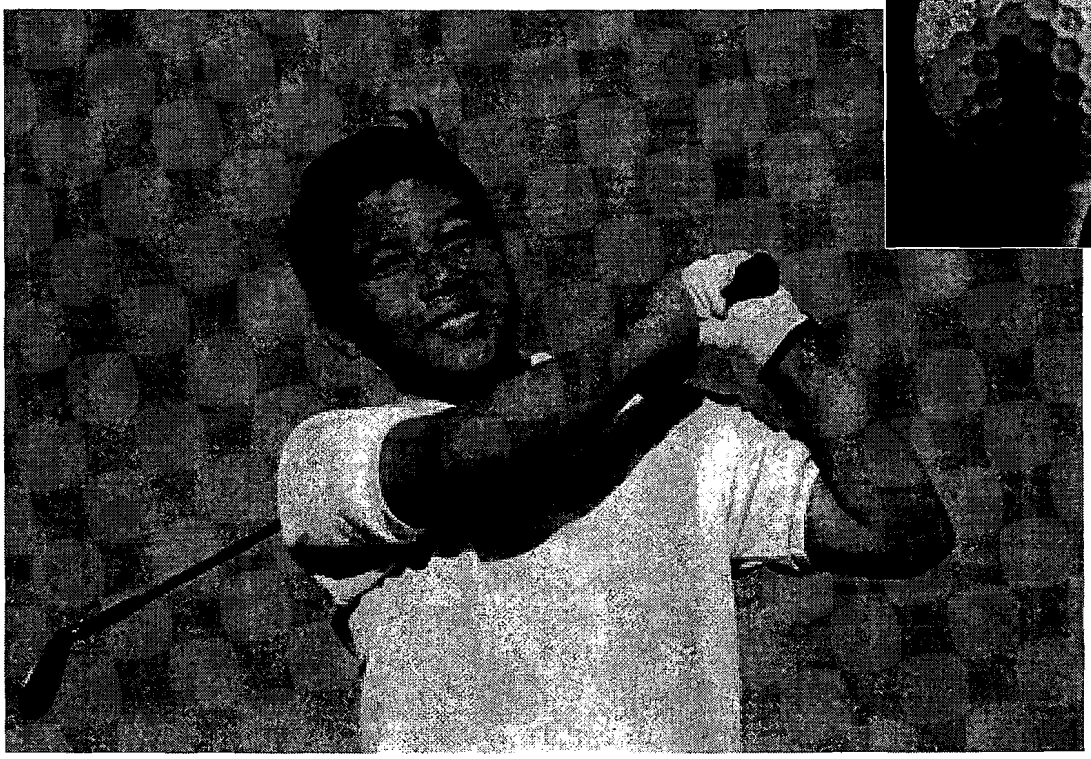
2. What do you observe about the angles that meet at a point in the tiling?

ACE **Homework starts on page 62.**

Did You Know

One of the leading golf ball manufacturers developed a pattern using hexagons for golf balls. They claim it is the first design to cover 100% of the surface area of a golf ball. This pattern of mostly hexagons almost eliminates flat spots found on typical golf balls, which interfere with their performance. This new design produces a longer, better flight for the golf ball.





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3.4 Exterior Angles of Polygons

TEKS / TAKS 6(6)B Identify relationships involving angles in triangles. 6(8)C Measure angles. 6(11)A Identify mathematics in activities outside of school.

An angle *inside* a polygon, formed by the polygon's sides, is an **interior angle.** By extending a side of a polygon, you can make an **exterior angle,** which is *outside* the polygon. Extending a side of the polygon forms one ray of the exterior angle.

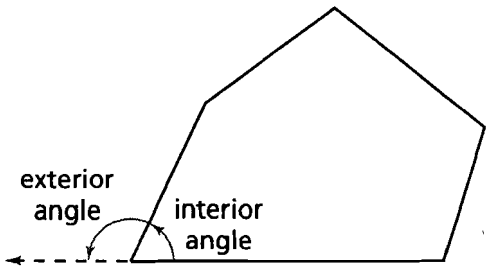


Figure 1 shows the exterior angles made by extending sides as you move counterclockwise around the polygon. Figure 2 shows the exterior angles formed by extending sides as you move clockwise around the polygon.

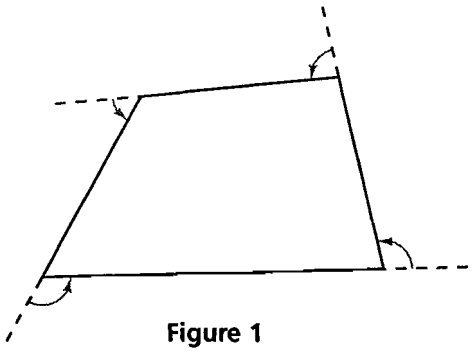


Figure 1
Exterior angles as you move counterclockwise.

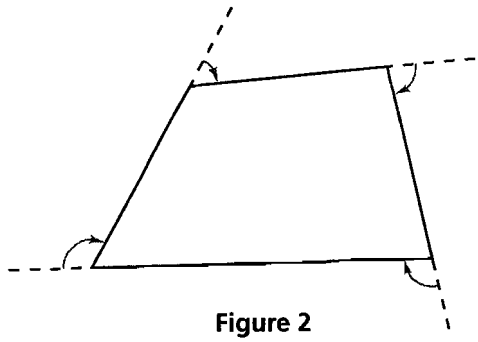
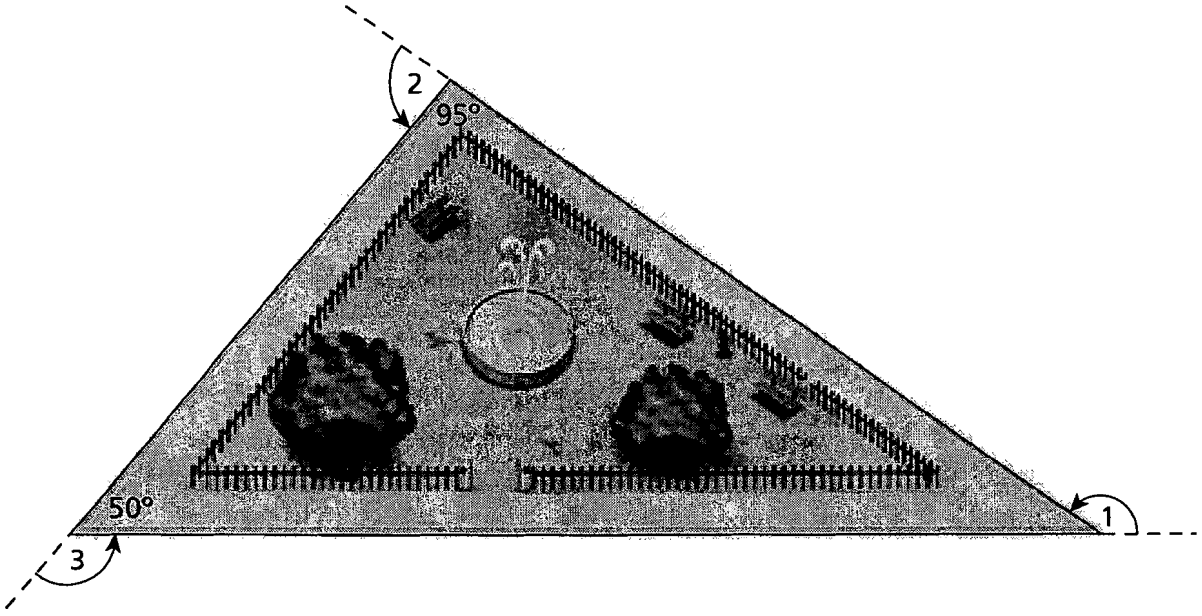


Figure 2
Exterior angles as you move clockwise.

Problem 3.4 Exterior Angles of Polygons

A skateboarder is skating on a triangular path around a park. In the diagram below, each segment of the path has been extended to show the angle of turn the in-line skater makes as she turns the corner. Each of these angles is an exterior angle of the triangle.



A. **1.** What are the measures of the interior angles of the triangle?

2. What is the measure of angle 1?

3. What are the measures of angle 2 and angle 3?

B. Suppose the skateboarder skates once around the park counterclockwise, turning each corner exactly once. What is the sum of the angles through which she turns?

C. **1.** Draw another triangle and mark the exterior angles going in one direction around the triangle.

2. Measure the exterior angles and find the sum.

3. Compare the exterior angle sum of your triangle to the sum you found for the triangle in Question B.

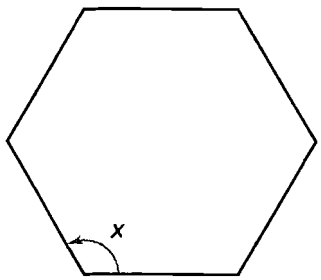
4. Can you predict the exterior angle sum for another triangle? Explain.

CE **Homework starts on page 62.**

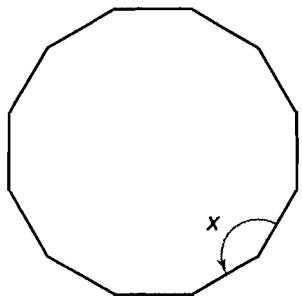
Applications

1. Without measuring, find the measures of the angle labeled x in each regular polygon.

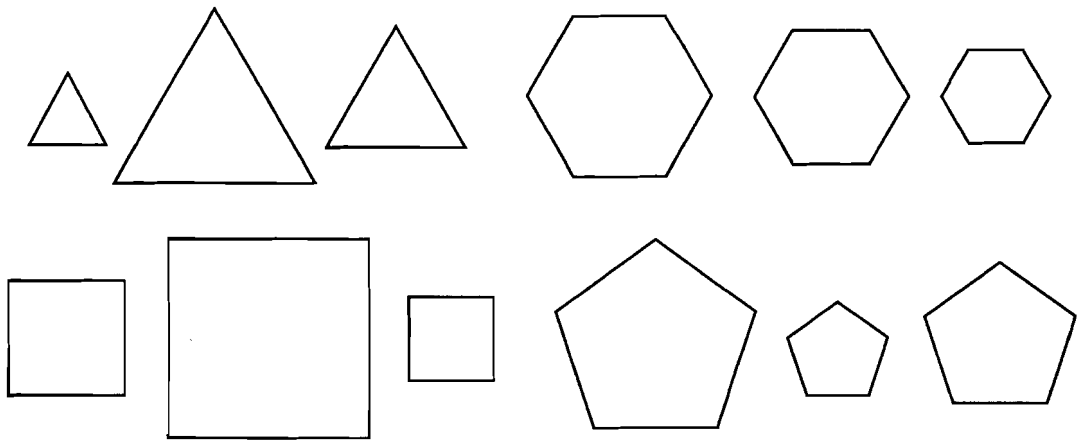
a.



b.

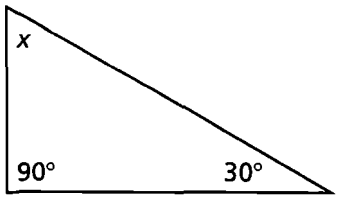


2. Below are sets of regular polygons of different sizes. Does the length of a side of a regular polygon affect the sum of the interior angle measures? Explain.

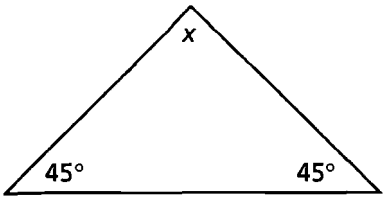


For Exercises 3–9, find the measure of each angle labeled x.

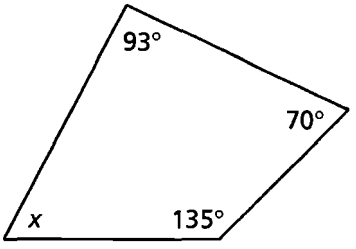
3.



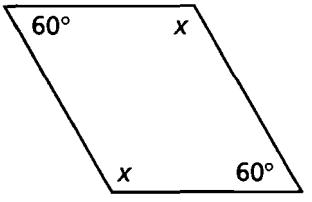
4.



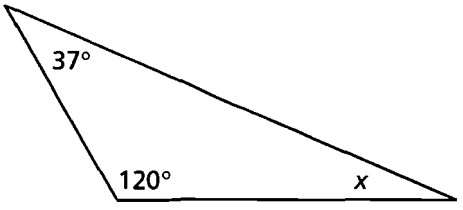
5.



6. This figure is a parallelogram.



7.



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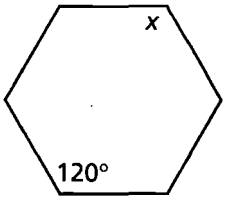
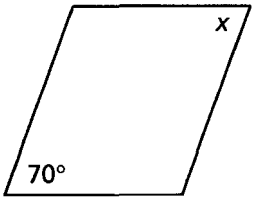
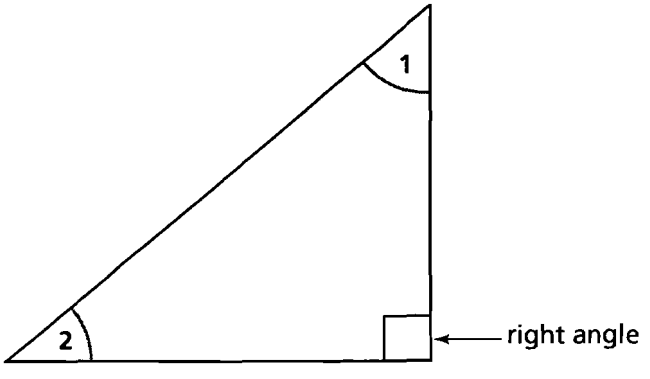
8. This figure is a regular hexagon.



9. This figure is a parallelogram.



10. A **right triangle** has one right angle and two acute angles. Without measuring, find the sum of the measures of the two acute angles. Explain.



11. This figure is a regular dodecagon. A dodecagon has 12 sides.

a. What is the sum of the measures of the angles of this polygon?

b. What is the measure of each angle?

c. Can copies of this polygon be used to tile a surface? Explain.

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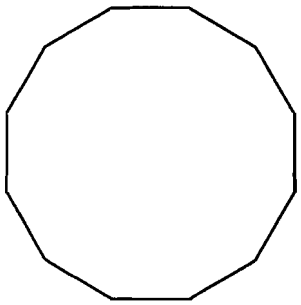


12. Multiple Choice Which of the following will tile a plane?

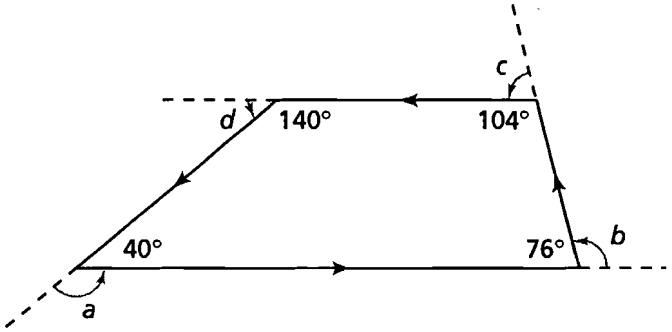
A. regular heptagon and equilateral triangle

B. square and regular octagon

C. regular pentagon and regular hexagon

D. regular hexagon and square

13. Suppose an in-line skater skates around a park that has the shape of a quadrilateral. Suppose he skates once around the quadrilateral, turning each corner exactly once. What is the sum of the angles through which he turns?

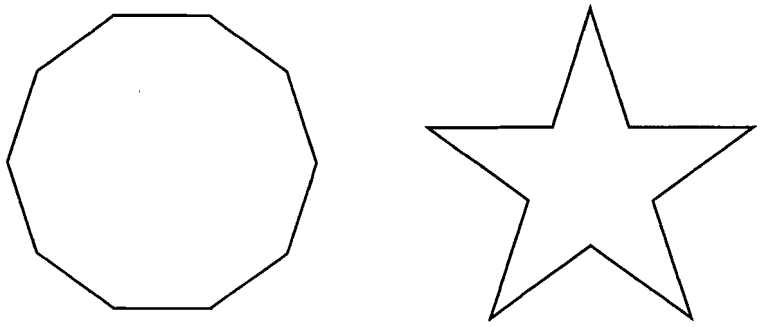


14. a. Suppose an in-line skater skates around a park that has the shape of a regular pentagon. If he skates once around the pentagon, turning each corner exactly once, what is the sum of the angles through which he turns?

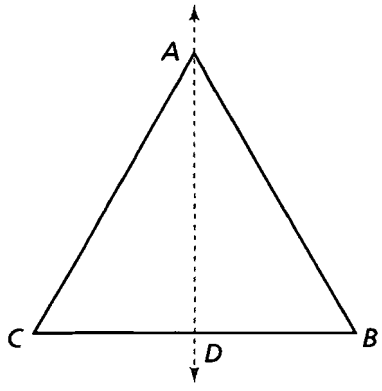
b. How many degrees will the skater turn if he skates once around a regular hexagon? A regular octagon? A regular polygon with N sides? Explain.

Connections

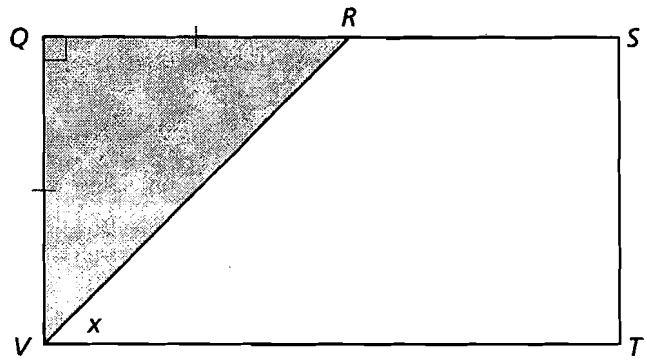
15. A regular decagon and a star are shown below. Measure the angles inside the star to find the angle sum of the star. Compare your results to the angle sum for a regular decagon.



16. In the diagram below, the dashed line is a line of symmetry for the equilateral triangle. Examine the two smaller triangles that are formed by the dashed line. What do you know about the angles and the line segments of triangles ABD and ACD? Give reasons to support the relationships you find.



17. Multiple Choice Figure $QSTV$ is a rectangle. The lengths QR and QV are equal. What is the measure of angle x?



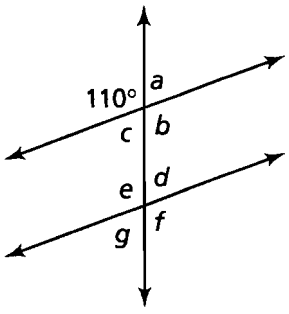
F. 20° **G.** 45°

H. 90° **J.** 120°

18. Choose a non-rectangular parallelogram from your Shapes Set or draw one of your own. Try to fit copies of the parallelogram exactly around a point. Sketch a picture to help explain what you found.

19. Choose a scalene triangle from your Shapes Set or draw one of your own. Try to fit copies of your triangle exactly around a point. Sketch a picture to help explain what you found.

20. In the diagram below, two parallel lines are cut by a transversal. Use what you learned in Investigation 2 to find the missing angle measures.



Extensions

21. a. Complete this table about regular polygons and look for a pattern.

Regular Polygons

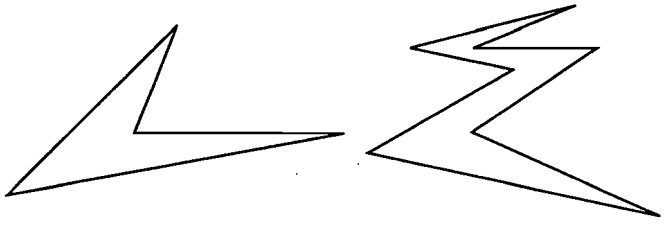
Number of Sides	Measure of Interior Angle
4	$\frac{1}{2}$ of 180°
6	$\frac{2}{3}$ of 180°
8	$\frac{3}{4}$ of 180°
10	▩

b. Does this pattern continue? Explain.

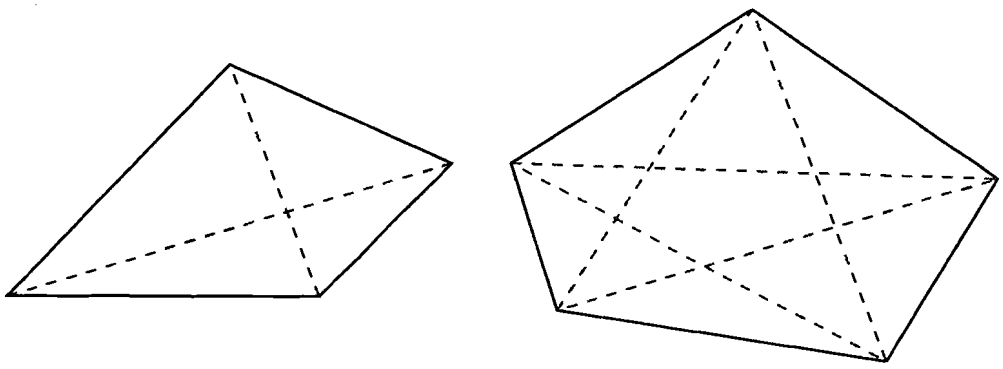
c. Is there a similar pattern for regular polygons with odd numbers of sides?

22. Kele claims that the angle sum of a polygon that he has drawn is 1,660°. Can he be correct? Explain.

23. Look at the polygons below. Does Tia's method of finding the angle sum (Problem 3.2) still work? Does Cody's method also still work? Can you still find the angle sum of the interior angles without measuring? Explain.



24. Below are a quadrilateral and a pentagon with the diagonals drawn from all the vertices.



a. How many diagonals does the quadrilateral have? How many diagonals does the pentagon have?

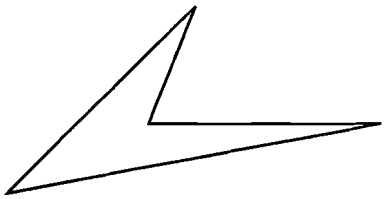
b. Find the total number of diagonals for a hexagon and for a heptagon.

c. Copy the table below and record your results from parts (a) and (b). Look for a pattern relating the number of sides and the number of diagonals to complete the table.

Number of Sides	4	5	6	7	8	9	10	11	12
Number of Diagonals									

d. Write a rule for finding the number of diagonals for a polygon with N sides.

25. Would a quadrilateral like the one below tile a plane? Explain.



Mathematical Reflections 3

In this investigation, you explored patterns for angle sums of polygons. You also investigated how the interior angle measures of a polygon determine whether copies of the polygon will fit exactly around a point. The questions below will help you summarize what you have learned.

Think about your answers to these questions, and discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **a.** What is the angle sum of a triangle? A quadrilateral? A hexagon? A polygon with N sides?

   b. Describe how you can find the measure of each interior angle of a regular polygon.

   c. As the number of sides in a regular polygon increases, what happens to the measure of an interior angle?

2. Describe how you can find the sum of the measures of the exterior angles of a polygon.

Unit Project What's Next?

What information about shapes can you add to your *Shapes and Designs* project?

Name _____ Date _____ Class _____

Labsheet 3.2

Angle Sums

Polygon	Number of Sides	Angle Sum Using Tia's Method	Angle Sum Using Cody's Method	Angle Sum
Triangle	3			
Quadrilateral	4			
Pentagon	5			
Hexagon	6			
Heptagon	7			
Octagon	8			
Nonagon	9			
Decagon	10			