

Exercise Set #7

1. What is the distance between the points $4 + i/2$ and $4 + 5i$ in the Poincaré upper half plane, H ?
2. What is the distance between the points $-2 + 2i$ and $-2 + 7i$ in the Poincaré upper half plane, H ?
3. Prove that the dilation $\delta_\lambda(x, y) = (\lambda x, \lambda y)$ preserves the Poincaré arclength element.
4. Let $P = 4 + 4i$ and $Q = 5 + 3i$. Find M and N , the endpoints of the Poincaré line through P and Q .
5. Let $P = 12i$ and $Q = 7 + 5i$. Find M and N , the endpoints of the Poincaré line through P and Q .

6. In the upper half plane model, H , carefully draw the asymptotic triangle with vertices i , $1 + i$, and 1 . Is the map

$$\gamma = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

an isometry of H ? In the same diagram, carefully draw the image of the asymptotic triangle under the action of γ .

7. In the upper half plane model, H , carefully draw the asymptotic triangle with vertices i , $-1 + i$, and $1 + i$. In the same diagram, carefully draw the image of this triangle under the isometry

$$\gamma = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

8. Let $P = \frac{8+i}{13}$, $Q = \frac{13+i}{20}$, and $\gamma = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$. What are γP and γQ ? Sketch P , Q and their images. Is γ an isometry? Why? Use all of this information to find the distance between P and Q in H .

9. Let $P = 2 + 4i$ and $Q = \frac{6+4i}{3}$ be two points in the upper half plane, H . Let

$$\gamma = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}.$$

What are γP and γQ ? What is the Poincaré distance from P to Q in H .

10. Suppose that T is a fractional linear transformation such that $T(1) = 1$, $T(0) = 0$, and $T(\infty) = \infty$. Prove that T is the identity map. That is, show that $T(z) = z$ for all z .