Problem Set 1

1. Let $F(x,y) = f(h(x), g(y), k(x,y))$, where g, h, and k are differentiable function of the variables indicated. Find $\partial F/\partial x$ and $\partial F/\partial y$.

2. Let $f(x,y,z) = x^2 + \sin(yz) - 3$. Find $D(f/x)(1, \pi, -1)$ and $D(x^2 y f)(2, 0, 1)$.

3. Let $f\colon \mathbf{R}^3 \to \mathbf{R}$ and $\sigma\colon \mathbf{R} \to \mathbf{R}^3$ be given by $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ and $\sigma(t) = (\cos(t), \sin(t), 1)$. Compute $D(f\circ\sigma)(t)$ and $D(f\circ\sigma)(0)$.

4. Let $f(x,y) = x^3 y$, where $x^3 + tx = 8$ and $ye^y = t$. Find $\dfrac{\partial f}{\partial t}(0)$.
   Hint: You must use the chain rule.

5. Express the equation of the paraboloid $z = 4 - x^2 - y^2$ and the equation of the plane $x + 2y - z = 0$ in both cylindrical coordinates and spherical coordinates.

6. Describe the surface whose equation in spherical coordinates is $\rho = 2\sin\phi$.

7. Represent the vector field $\mathbf{F}(x,y,z) =< x^2 - y^2, 0, -1 >$ in both cylindrical and spherical coordinate systems.

8. Parabolic cylindrical coordinates (u, v, z) are given by

   $$x = \frac{1}{2}(u^2 - v^2), \qquad y = uv, \qquad z = z,$$

   where $-\infty < u < \infty$, $v \geq 0$, and $-\infty < z < \infty$. Describe the coordinate curves. Find the unit coordinate vectors. Express the vector field $\mathbf{F}(x,y,z) =< x, y^2, -2y >$ in the coordinate system (u, v, z).