Quiz 10 — 12/08/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. A toucan population grows logistically with constant growth $k = 0.4$ per year in a forest with carrying capacity of 1000 toucans. Write the differental equation that describes the rate of change of the population.

The logistic differential equation is given by the formula: $\frac{dy}{dt} = k \cdot y\left(1 - \frac{y}{A}\right)$ where k is the growth constant and $A > 0$ is the carrying capacity. For $k = 0.4$ and $A = 1000$, we get:

$$\frac{dy}{dt} = 0.4 \cdot y\left(1 - \frac{y}{1000}\right)$$

2. Solve the logistic equation:

$dy/dx = 0.4y(1 - y/1000)$ with the initial condition $y(0) = 100$.

$$\frac{dy}{y\left(1-\frac{y}{1000}\right)} = 0.4\,dx \Rightarrow \int \frac{dy}{y\left(1-\frac{y}{1000}\right)} = \int 0.4\,dx \Rightarrow$$

$$\int \left(\frac{1}{y} - \frac{1}{y-1000}\right) dy = \int 0.4\,dx \Rightarrow \ln|y| - \ln|y-1000| = 0.4x + C \Rightarrow \ln\left|\frac{y}{y-1000}\right| = 0.4x + C \Rightarrow$$

$$\frac{y}{y-1000} = \pm e^{0.4x+C} = \underbrace{\pm e^{C}}_{=C} \cdot e^{0.4x} = C \cdot e^{0.4x} \Rightarrow$$

$$y = (y-1000)\, C \cdot e^{0.4x} \Rightarrow y\left(1 - C \cdot e^{0.4x}\right) = -1000\, C \cdot e^{0.4x}$$

$$\Rightarrow \boxed{y = \frac{-1000 \cdot C \cdot e^{0.4x}}{1 - C \cdot e^{0.4x}}}$$

For C: $100 = \frac{-1000 \cdot C \cdot e^{0.4 \cdot 0}}{1 - C \cdot e^{0.4 \cdot 0}}$

$$1 - C = -10C \Rightarrow \boxed{C = -\frac{1}{9}}$$

Hence, the solution is: $y = \frac{-1000 \cdot \left(-\frac{1}{9}\right) \cdot e^{0.4x}}{1 - \left(-\frac{1}{9}\right) \cdot e^{0.4x}}$