

MA 114 Worksheet #20: Arc length & Surface area

1.
 - (a) Write down the formula for the arc length of a function $f(x)$ over the interval $[a, b]$ including the required conditions on $f(x)$.
 - (b) Write down the formula for the surface area of a solid of revolution generated by rotating a function $f(x)$ over the interval $[a, b]$ around the x -axis. Include the required conditions on $f(x)$.
 - (c) Write down the formula for the surface area of a solid of revolution generated by rotating a function $f(x)$ over the interval $[a, b]$ around the y -axis. Include the required conditions on $f(x)$.
2. Find an integral expression for the arc length of the following curves. Do **not** evaluate the integrals.
 - (a) $f(x) = \sin(x)$ from $x = 0$ to $x = 2$.
 - (b) $f(x) = x^4$ from $x = 2$ to $x = 6$.
 - (c) $x^2 + y^2 = 1$
3. Find the arc length of the following curves.
 - (a) $f(x) = x^{3/2}$ from $x = 0$ to $x = 2$.
 - (b) $f(x) = \ln(\cos(x))$ from $x = 0$ to $x = \pi/3$.
 - (c) $f(x) = e^x$ from $x = 0$ to $x = 1$.
4. Set up a function $s(t)$ that gives the arc length of the curve $f(x) = 2x + 1$ from $x = 0$ to $x = t$. Find $s(4)$.
5. Compute the surface areas of revolution about the x -axis over the given interval for the following functions.
 - (a) $y = x$, $[0, 4]$
 - (b) $y = x^3$, $[0, 2]$
 - (c) $y = (4 - x^{2/3})^{3/2}$, $[0, 8]$
 - (d) $y = e^{-x}$, $[0, 1]$
 - (e) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $[1, e]$
 - (f) $y = \sin x$, $[0, \pi]$
 - (g) Find the surface area of the torus obtained by rotating the circle $x^2 + (y - b)^2 = r^2$ about the x -axis.
 - (h) Show that the surface area of a right circular cone of radius r and height h is $\pi r \sqrt{r^2 + h^2}$.
Hint: Rotate a line $y = mx$ about the x -axis for $0 \leq x \leq h$, where m is determined by the radius r .