$\frac{21}{21}$

Written HW 3

2/9/2017

MA 322

1) For each transformation T find the standard matrix, A and find m and n so that $T: \mathbb{R}^n \to \mathbb{R}^m$

(a) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x \\ -y \end{bmatrix}$ m×n = rows x columns $T: \mathbb{R}^n \to \mathbb{R}^m$

$\frac{6}{6}$

In this question, the standard matrix A would have to be of size 3x2 in order to get the solution given above, knowing T is of size 2x1.

A generalized look at matrix A is below:

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_1 \\ a_3 \\ a_5 \end{bmatrix} x + \begin{bmatrix} a_2 \\ a_4 \\ a_6 \end{bmatrix} y = \begin{bmatrix} x+y \\ x \\ -y \end{bmatrix}$$

Seeing this, we can see that $T: \mathbb{R}^n \to \mathbb{R}^m$ is equivalent to $T: \mathbb{R}^2 \to \mathbb{R}^3$ The standard matrix A is of size 3x<u>2</u>, while the transformation is of size <u>3</u>x1.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}$$ in order to achieve solution above.

(b) $T(x_1, x_2, x_3, x_4) = (x_1 + 5x_2,\ x + 2x_3,\ x_2 - x_3)$

The question can be rewritten as:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 5x_2 \\ x_1 + 2x_3 \\ x_2 - x_3 \end{bmatrix}$$

The standard matrix A must be of size 3x<u>4</u>, for the transformation to be $\mathbb{R}^3$. Therefore the solution for $T: \mathbb{R}^n \to \mathbb{R}^m$ would be $T: \mathbb{R}^4 \to \mathbb{R}^3$. The standard matrix A would be as follows:

$$A = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Taking the transformation given of this matrix can be written as follows:

$$\begin{bmatrix} 1 & 5 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 + 5x_2 \\ x_1 + 2x_3 \\ x_2 - x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} x_4 = \begin{bmatrix} 1x_1 + 5x_2 \\ 1x_1 + 2x_3 \\ 1x_2 - 1x_3 \end{bmatrix}$$

This was found by knowing $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ then using Same method as problem 1a.

$$\begin{bmatrix} a_1 & a_4 & a_7 & a_{10} \\ a_2 & a_5 & a_8 & a_{11} \\ a_3 & a_6 & a_9 & a_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

2) An affine transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ has the form $T(x) = Ax + b$ with $A$ an $m \times n$ matrix, and $b$ in $\mathbb{R}^m$.
Show $T$ is not a linear transformation if $b \neq 0$.

5/5

If $T$ is a transformation:

$$T(0) = 0$$

In this problem

$T(0) = A(0) + b$ where it would equal $b$, but if $b$ does not equal zero then $T$ is not a linear transformation.

3) Suppose $T$ is a linear transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$.
Give a relationship between $m$ and $n$.

6/6

(a) if $T$ is onto?

Given that $T$ is a linear transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ then $m > n$ is a relationship where $T$ cannot be onto. This is given from the definition that states a mapping $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be onto $\mathbb{R}^m$ if each $b$ in $\mathbb{R}^m$ is the image of at least one $x$ in $\mathbb{R}^n$.

$\Rightarrow m \leq n$ ONTO

(b) if T is one-to-one?

A relationship between m and n for a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$

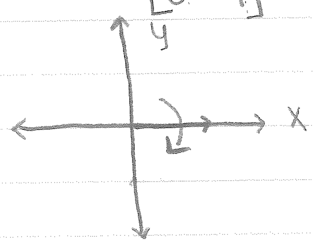
Given from a definition that states that each $b$ in $\mathbb{R}^m$ is the image of <u>at most</u> <u>one</u> $x$ in $\mathbb{R}^n$, we can note that a linear transformation T is not one-to-one if $m < n$. $\Rightarrow m \geq n$ ONE-TO-ONE

4) Let T be a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ which reflects points through the line $y = x$. Find the standard matrix associated to this linear transformation.

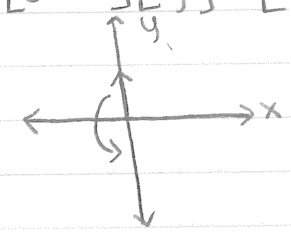
If T is the reflection about the x-axis:

$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ then $Ax = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$

4/4




$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$




$\vec{e}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

Next we know that T is the reflection through the line $y = x$, therefore the matrix would be as follows:

$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Therefore we can use the properties available to solve

$T_B(T_A(x)) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix} \longrightarrow T_B(T_A(x)) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Giving the standard matrix of

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$