SOLUTIONS

1. Use the tables to find the inverse Laplace transform of $F(s) = \frac{1}{s-7} + \frac{e^{-2s}}{s-4} + 2$.

For the middle term: if $G(t) = \frac{1}{s-4}$, then $g(t) = e^{4t}$

For $\frac{e^{-2s}}{s-4}$, we have $c = 2$: $g(t-c) = g(t-2)$
$= e^{4(t-2)}$

So, the inverse of the middle term is $u_2(t)\, e^{4(t-2)}$.

For the final answer (all three terms):

$$\boxed{f(t) = e^{7t} + u_2(t)\, e^{4(t-2)} + 2\,\delta(t)}$$

2. Solve the initial value problem $y'' + 4y' + 13y = 3\delta(t-3)$; $y(0) = 0$, $y'(0) = 0$.

apply the Laplace transform to both sides:

$$s^2 Y(s) - s y(0) - y'(0) + 4(sY(s) - y(0)) + 13 Y(s) = 3e^{-3s}$$

Apply initial conditions:

$$s^2 Y(s) + 4s Y(s) + 13 Y(s) = 3e^{-3s}$$

Solve for $Y(s) = \frac{3e^{-3s}}{s^2+4s+13}$ ← doesn't factor, so complete the square:

$$= e^{-3s} \cdot \frac{3}{s^2+4s+4+9} = \underset{c=3}{e^{-3s}} \cdot \underbrace{\frac{3}{(s+2)^2+3^2}}$$

If $F(s) = \frac{3}{(s+2)^2+3^2}$, Then (table) $f(t) = e^{-2t}\sin(3t)$,

So $f(t-c) = f(t-3) = e^{-2(t-3)}\sin(3(t-3))$

Thus $\boxed{y(t) = u_3(t)\, e^{-2(t-3)}\sin(3(t-3))}$