MA 214—04 Review Problems to help with Exam 1

These problems should help prepare for the first exam. This collection is NOT comprehensive! Look over all of the previous worksheets, homework, class examples and the first quiz.

1. Solve the initial value problems.
   a. $y' = x + y$; $y(0) = 1$

   b. $y' = xy - 3y$; $y(1) = 1$

   c. $(\cos y + y\cos x)\,dx + (\sin x - x\sin y)\,dy = 0$; $y(0) = -1$

   d. $y'' - 2y' - 8y = 0$; $y(0) = 3, y'(0) = \pi$.

   e. $4y'' + 4y' + y = 0$; $y(0) = 1,\ y'(0) = 3$.

2. Consider the differential equation $(x + y) + (ax + 2y)\dfrac{dy}{dx} = 0$.
   a. Find a value for a which makes the equation exact.
   b. For your choice of a, solve the equation. Give an implicit general solution.
   c. Find an explicit solution satisfying $y(0) = 1$.

3. Determine the largest interval where the following differential equations will have a unique solution.
   a. $(\cos t)\,y' - (\sin t)\,y = 3t\cos t$; $y(2\pi) = 0$.

   b. $(t^2 - 81)\,y' + 5e^{3t}y = \sin t$; $y(10) = 10\pi$.

4. A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the the tank has a concentration of $\frac{1}{5}(1+\cos t)$ lbs/gal. If a well-mixed solution leaves the tank at a rate of 6 gal/hr, we want to know how much salt is in the tank when it overflows.
   a. Let $Q(t)$ be the pounds of salt in the water at time t. Write a differential equation which models this situation.
   b. Your equation should be first-order linear. Find an integrating factor.
   c. (Save for the end): Solve the equation and answer the question.

5. Consider the equation $y'' + 4y = 0$.
   a. Verify $y_1 = \cos 2t$ and $y_2 = \sin 2t$ are solutions to the equation, and verify that they are linearly independent.
   b. Explain why we expect for $y = C_1 y_1 + C_2 y_2$ to be the general solution.

6. Consider the equation $yy'' + (y')^2 = 0$ for $t > 0$.
   a. Verify $y_1 = 1$ and $y_2 = t^{1/2}$ are solutions to the equation, and that they are linearly independent.
   b. Show that $y = 1 + t^{1/2}$ is NOT a solution to the equation. Explain why we do ***not*** expect in this case for $y = C_1 y_1 + C_2 y_2$ to be the general solution.