SOLUTIONS

MA 214—04 2nd order linear homogeneous, including those with complex roots

1. Write the general solution for each equation.

   a. $y''-2y'+2y=0$

   $r^2-2r+2=0$

   $r=\frac{2\pm\sqrt{2^2-4(1)(2)}}{2(1)}=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm\sqrt{-4}}{2}=\frac{2\pm 2i}{2}=1\pm i$, complex roots

   $\boxed{y(t)=c_1e^{t}\cos(t)+c_2e^{t}\sin(t)}$

   b. $y''+2y'-8y=0$

   $r^2+2r-8=0$

   $(r+4)(r-2)=0$

   $r=-4,2$, distinct real roots

   $\boxed{y(t)=c_1e^{-4t}+c_2e^{2t}}$

   c. $y''-10y'+25y=0$

   $r^2-10r+25=0$

   $(r-5)^2=0$

   $r=5$, repeated roots

   $\boxed{y(t)=c_1e^{5t}+c_2te^{5t}}$

   d. $4y''+9y=0$

   $4r^2+9=0$

   $4(r^2+9/4)=0$

   $r^2+9/4=0$

   (can also find w/ quadratic formula) $(r-\frac{3}{2}i)(r+\frac{3}{2}i)=0$

   $r=\pm\frac{3}{2}i$, complex roots

   $\boxed{y(t)=c_1\cos\left(\frac{3}{2}t\right)+c_2\sin\left(\frac{3}{2}t\right)}$

2. Solve the initial value problem $y''+4y=0$; $y(0)=0$; $y'(0)=1$.

Characteristic Equation: $r^2+4=0$ or: $r^2=-4$

$(r+2i)(r-2i)=0$

So, $r=\pm 2i$

General Solution: $y(t)=c_1\cos(2t)+c_2\sin(2t)$

$0=y(0)=c_1\cos(0)+c_2\sin(0)$

$\underline{0=c_1}$

So, $y(t)=c_2\sin(2t)$

$y'(t)=2c_2\cos(2t)$

$1=y'(0)=2c_2\cos(0)$

so, $1=2c_2$ and $\underline{c_2=1/2}$.

The solution to the IVP is

$\boxed{y(t)=\frac{1}{2}\sin(2t)}$