SOLUTIONS

MA 214—04 Method of Undetermined Coefficients

Find the general solution for each nonhomogeneous equation following these steps:

(a) Find the solution to the associated homogeneous equation.
(b) Use the method of undetermined coefficients to find a particular solution.
(c) Write the general solution as the sum of your answers from parts (a) and (b).

1. $y'' + y' - 2y = 2x$

(a) Associated homogeneous eqn: $y'' + y' - 2y = 0$

$r^2 + r - 2 = 0$

$(r+2)(r-1) = 0$ $\quad y_h(x) = c_1 e^{-2x} + c_2 e^{x}$

$r = -2, 1$

(b) Guess: $y_p(x) = ax + b$

$y_p'(x) = a$

$y_p''(x) = 0$

Plugging into the eqn:

$0 + a - 2(ax+b) = 2x$

$-2ax + a - 2b = 2x$

So, we must have:

$-2a = 2 \Rightarrow \underline{a = -1}$

$a - 2b = 0$

$-1 - 2b = 0$

$-1 = 2b$

$\Rightarrow \underline{b = -1/2}$

So, $y_p(x) = -x - 1/2$ is a particular soln.

(c) $\boxed{y(x) = c_1 e^{-2x} + c_2 e^{x} - x - 1/2}$

2. $y'' - 2y' - 3y = 3e^{2t}$

(a) $y'' - 2y' - 3y = 0$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$r = -1, 3$

$y_h(x) = c_1 e^{-t} + c_2 e^{3t}$

(b) Guess: $y_p(t) = Ae^{2t}$

$y_p'(t) = 2Ae^{2t}$

$y_p''(t) = 4Ae^{2t}$

Plugging in:

$4Ae^{2t} - 2(2Ae^{2t}) - 3(Ae^{2t}) = 3e^{2t}$

$-3Ae^{2t} = 3e^{2t} \Rightarrow -3A = 3$, so $A = -1$

So $y_p(t) = -e^{2t}$ is a solution.

(c) $\boxed{y(t) = c_1 e^{-t} + c_2 e^{3t} - e^{2t}}$

3. $y'' + 2y' + 5y = 3\sin 2t$

(a) $y'' + 2y' + 5y = 0$

$r^2 + 2r + 5 = 0$

$r = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{4-20}}{2} = \dfrac{-2 \pm \sqrt{-16}}{2}$

$= \dfrac{-2 \pm 4i}{2} = -1 \pm 2i$

So, $y_h(t) = c_1 e^{-t}\cos(2t) + c_2 e^{-t}\sin(2t)$

(b) Guess: $y_p(t) = A\cos(2t) + B\sin(2t)$

$y_p'(t) = -2A\sin(2t) + 2B\cos(2t)$

$y_p''(t) = -4A\cos(2t) - 4B\sin(2t)$

Plugging in: $\underbrace{-4A\cos(2t) - 4B\sin(2t)}_{y''} \overbrace{-4A\sin(2t) + 4B\cos(2t)}^{2y'}$

$\underbrace{+5A\cos(2t) + 5B\sin(2t)}_{5y} = 3\sin(2t)$

So, $\cos(2t)(-4A + 4B + 5A)$

$+ \sin(2t)(-4B - 4A + 5B) = 3\sin(2t)$

So, we have:

$A + 4B = 0$

and $B - 4A = 3$

continued! →

We have $A = -4B$,

so we have $B - 4(-4B) = 3$

$B + 16B = 3$

$17B = 3$

$B = 3/17$.

Then $A = -4B = -12/17$

So, we have $y_p(t) = \frac{-12}{17}\cos(2t) + \frac{3}{17}\sin(2t)$ is a particular solution.

1c) $\boxed{y(t) = c_1 e^{-t}\cos(2t) + c_2 e^{-t}\sin(2t) - \frac{12}{17}\cos(2t) + \frac{3}{17}\sin(2t)}$