SOLUTIONS

MA 214—04 Warm-up / Review

1. Find the general solution for each equation.

a. $y''+2y'=0$

Characteristic Equation: $r^2+2r=0$

$r(r+2)=0$

$r=0, -2$ $\quad y(t)=c_1e^{0t}+c_2e^{-2t}=\boxed{c_1+c_2e^{-2t}}$

b. $y''+4y=0$

$r^2+4=0$

$(r+2i)(r-2i)=0$

$r=\pm 2i$

$y(t)=c_1e^{0t}\cos(2t)+c_2e^{0t}\sin(2t)$

$\boxed{y(t)=c_1\cos(2t)+c_2\sin(2t)}$

c. $y''-2y'+y=0$

$r^2-2r+1=0$

$(r-1)^2=0$

$r=1$

$\boxed{y(t)=c_1e^t+c_2te^t}$

2. Find and simplify the Wronskian of $y_1=\cos(3t)$ and $y_2=\sin(3t)$.

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos(3t) & \sin(3t) \\ -3\sin(3t) & 3\cos(3t) \end{vmatrix}$$

$$=3\cos^2(3t)+3\sin^2(3t)=3(\cos^2(3t)+\sin^2(3t))=3\cdot 1=\boxed{3}$$

3. Find the general solution to $y''+2y'=3+4\sin(2t)$. (Hint: compare to 1a above.)

Note: The left-hand side is the same as in 1(a).

Guess: $y_p=\underbrace{A\cos(2t)+B\sin(2t)}_{\text{trig in right-hand side}}+\underbrace{Ct}$

polynomial of degree 0 on right & constants are part of the homogeneous solution from 1(a), so we multiply by t

$y_p'=-2A\sin(2t)+2B\cos(2t)+C$

$y_p''=-4A\cos(2t)-4B\sin(2t)$

Plugging in to the eqn:

$$\underbrace{-4A\cos(2t)-4B\sin(2t)}_{y_p''}\underbrace{-4A\sin(2t)+4B\cos(2t)+2C}_{2y_p'}=3+4\sin(2t)$$

continued →

So, $\cos(2t)(-4A+4B)+\sin(2t)(-4B-4A)+2c=3+4\sin(2t)$

Therefore, we have:

$\cos(2t)(-4A+4B)=0$

$\Rightarrow -4A+4B=0$

$\sin(2t)(-4B-4A)=4\sin(2t)$

$\Rightarrow -4A-4B=4$

$2c=3$

$\Rightarrow \boxed{c=\frac{3}{2}}$

$-4A+4B=0$

and $-4A-4B=4$.

$\Rightarrow$ So, $4B=4A \Rightarrow B=A$.

Then $-4A-4A=4$

$-8A=4$

$\boxed{A=-1/2=B}$

So, $y_p(t)=-\frac{1}{2}\cos(2t)-\frac{1}{2}\sin(2t)+\frac{3}{2}t$

Therefore, the general solution is:

$$\boxed{y(t)=c_1+c_2e^{-2t}-\frac{1}{2}\cos(2t)-\frac{1}{2}\sin(2t)+\frac{3}{2}t}$$