SOLUTIONS

MA 214—04 Second Order Homogenous with Constant Coefficients

1. Consider the equation $y''+4y'+3y=0$.
   a. Write the corresponding characteristic equation.
   b. Find the general solution.
   c. Find the solution satisfying $y(0)=2,\ y'(0)=-1$.

(a) $r^2+4r+3=0$

(b) $(r+3)(r+1)=0$

$r=-3,\ r=-1 \Rightarrow y_1=e^{-3t},\ y_2=e^{-t}$

$\boxed{y=c_1e^{-3t}+c_2e^{-t}}$

(c) If $x=0$, $y=2$: $\quad 2=c_1+c_2$

If $x=0$, $y'=-1$. $\quad y'=-3c_1e^{-3t}-c_2e^{-t}$

(take the deriv. to find y') $\quad -1=-3c_1-c_2$

Solve two equations + two unknowns.
We can add the equations: $\quad 1=-2c_1+0c_2$

$\Rightarrow c_1=-\frac{1}{2}$

Sub back in: $\quad 2=-\frac{1}{2}+c_2 \quad c_2=2+\frac{1}{2}=\frac{4}{2}+\frac{1}{2}=\frac{5}{2}$

$\boxed{y=-\frac{1}{2}e^{-3t}+\frac{5}{2}e^{-t}}$

2. Find a differential equation whose general solution is $y=C_1e^{2t}+C_2e^{-5t}$

using $r=2,\ r=-5$,

characteristic equation $(r-2)(r+5)=0$

$\Rightarrow r^2+3r-10=0$

corresponds to $\boxed{y''+3y'-10y=0}$