MA 214-004

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Great job!

Written Homework 1

1. Find the general solution to the differential equations. Implicit Solutions are fine.

a) $x\frac{dy}{dx}-2y=x^3e^{-2x}$

$$\frac{dy}{dx}-\frac{2}{x}y=x^2e^{-2x}$$

$$e^{\int p(x)\,dx}$$

$$e^{\int -\frac{2}{x}dx}$$

$$e^{-2\ln x}$$

$$e^{\ln x^{-2}}=\frac{1}{x^2}$$

$$\frac{1}{x^2}\frac{dy}{dx}+\left(-\frac{2}{x^3}\right)y=e^{-2x}$$

$$\frac{d}{dx}\left[\frac{y}{x^2}\right]=e^{-2x}$$

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$$\frac{y}{x^2}=\int e^{-2x}\,dx$$

$$\frac{y}{x^2}=-\frac{1}{2}e^{-2x}+C \checkmark$$

b) $x^2y^2\frac{dy}{dx}+1=y$

$$x^2y^2\frac{dy}{dx}=y-1$$

$u=y-1$ $\quad y=u+1$

$du=1$ $\quad (u+1)^2$

u^2+2u+1

$$\int\frac{y^2}{y-1}\,dy=\int\frac{1}{x^2}\,dx$$

$$\int\frac{u^2+2u+1}{u}\,du=\int x^{-2}\,dx$$

$$\int u+2+\frac{1}{u}\,du=\frac{x^{-1}}{-1}+c$$

$$\frac{1}{2}u^2+2u+\ln|u|=\frac{-1}{x}+c$$

$$\frac{1}{2}(y-1)^2+2(y-1)+\ln|y-1|=\frac{-1}{x}+c \checkmark$$

2. Consider the differential equation $2xy\,dx + (y^2 - x^2)\,dy = 0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad M \quad\quad\quad\quad N$

a) Show this equation is not exact

$$\Psi_{xy} = M_y = 2x$$
$$\Psi_{yx} = N_x = -2x$$

does not match, $M_y \neq N_x$,

$\therefore$ this equation is not exact ✓

b) We can find an integrating factor $\mu = e^{-\int p(y)dy}$ if $p(y) = \dfrac{M_y - N_x}{M}$ is a function of only y. Find and apply μ, and then show that the new equation is exact.

$$p(y) = \frac{M_y - N_x}{M}$$

$$p(y) = \frac{2x - (-2x)}{2xy}$$

$$p(y) = \frac{2}{y}$$

$$\mu = e^{-2\int \frac{1}{y}\,dy}$$

$$\mu = e^{-2\ln|y|}$$

$$\mu = e^{\ln y^{-2}}$$

$2xy^{-1}$

$$\mu = \frac{1}{y^2}$$

$$\overset{M}{\frac{2x}{y}}\,dx + \overset{N}{\left(1 - \frac{x^2}{y^2}\right)}\,dy = 0$$

$$M_y = \frac{-2x}{y^2}$$

$$N_x = \frac{-2x}{y^2}$$

$M_y = N_x$

$\therefore$ eqn is exact ✓

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c) Solve the new equation. Leave your general solution in implicit form.

$$\frac{2x}{y}\,dx + \left(1 - \frac{x^2}{y^2}\right) dy = 0$$ ✓

$$\psi = \int M\,dx$$
$$= \int \frac{2x}{y}\,dx$$
$$= \frac{2x^2}{2y} + h(y)$$
$$\psi = \frac{x^2}{y} + h(y)$$

$x^2 y^{-1}$

$$\psi_y = \frac{-x^2}{y^2} + h'(y) = N$$

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$$\frac{-x^2}{y^2} + h'(y) = 1 - \frac{x^2}{y^2}$$

$$\int h'(y) = \int 1\,dy$$
$$h(y) = y$$

$\psi = c$

$$\frac{x^2}{y} + y = c$$ ✓

3. Consider the differential equation $\frac{dy}{dx} = \sqrt{x+y}$ with initial condition $y(1) = 2$.

a) Use Euler's Method with step size $h = 0.2$ to estimate $y(1.6)$

	x	y	$f(x_n, y_n)$	$y_{n+1} = y_n + f(x_n, y_n)h$
$n=0$	1	2	$\sqrt{1+2} = \sqrt{3}$	$2 + \sqrt{3} \cdot .2 = 2.346$
$n=1$	1.2	2.346	$\sqrt{1.2+2.346} = 1.88$	$2.346 + (.2)(1.88) = 2.72$
$n=2$	1.4	2.72	$\sqrt{1.4+2.72} = 2.03$	$2.72 + (.2)(2.03) = 3.126$
$n=3$	1.6	$\boxed{3.126}$		

✓

~~[illegible]~~

Nevermind. ☺

Euler's method uses the slope at (x_n, y_n) to estimate

b) y_{n+1}. The fourth-order-Runge-Kutta method (RK4)

uses a weighted average of slopes in the interval $[x_n, x_{n+1}]$

to estimate y_{n+1}:

$$y_{n+1} = y_n + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h$$

where
$$m_1 = F(x_n, y_n)$$
$$m_2 = F(x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2} m_1)$$
$$m_3 = F(x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2} m_2)$$
$$m_4 = F(x_n + h, y_n + h m_3)$$

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Use this method with step size $h = 0.2$

to estimate $y(1.2)$

$$m_1 = F(1, 2) = \sqrt{1 + 2} = \sqrt{3} = 1.732$$
$$m_2 = F(x_n + \tfrac{0.2}{2}, y_n + \tfrac{0.2}{2}(1.732)) = F(1.1, 2.1732) = \sqrt{1.1 + 2.1732} = 1.809$$
$$m_3 = F(1.1, 2 + (.1)(1.809)) = F(1.1, 2.362) = \sqrt{1.1 + 2.362} = 1.86$$
$$m_4 = F(1 + .2, 2 + .2(1.86)) = F(1.2, 2.372) = \sqrt{1.2 + 2.372} = 1.89$$

$$y(1.2) = 2 + \frac{1}{6}(1.732 + 2(1.809) + 2(1.86) + 1.89)(.2)$$
$$= 2.365$$