MA 214—04 Logistic Growth and Euler's method

ANSWERS

1. Assuming $P > 0$, suppose a population develops according to the logistic equation $\frac{dP}{dt} = 0.03P - 0.00015P^2$ where t is measured in weeks.
   a. What is the carrying capacity?
   b. What is the value of the intrinsic growth constant k?
   c. For what values of P is the population increasing?
   d. Suppose the initial population is 150 critters. Write an expression for $P(t)$.

$$\frac{dP}{dt} = .03\left(1 - \frac{.00015}{.03}P\right)P = \boxed{.03\left(1 - \frac{P}{200}\right)P}$$

factor out .03 and P ("divide + flip")

(a) carrying capacity = $\boxed{200 \text{ critters}}$

(b) intrinsic growth constant = $\boxed{.03}$

(c) P is increasing where $\frac{dP}{dt}$ is positive. $1 - \frac{P}{200}$ is positive for $\boxed{0 < P < 200}$ (.03 and P are always positive)

(d) general solution $P = \dfrac{P_0 K}{P_0 + (K - P_0)e^{-kt}}$

using $P_0 = 150$, $K = 200$, $k = .03$, we have

$$P(t) = \frac{150(200)}{150 + (200-150)e^{-.03t}} = \frac{30000}{150 + 50e^{-.03t}} = \boxed{\frac{600}{3 + e^{-.03t}}}$$

all are correct; last is simplest.

2. Use Euler's method with step size 0.25 to compute the approximate y-values y_1, y_2 and y_3 of the solution of the initial-value problem $y' = 1 - 2x - 2y$, $y(1) = -1$.

	x_n	$y_n = y_{n-1} + y'(.25)$	$y' = 1-2x-2y$ (slope)
$n=0$	1	-1	$1 - 2(1) - 2(-1) = 1$
$n=1$	1.25	$-1 + 1(.25) = \boxed{-.75}$	$1 - 2(1.25) - 2(-.75) = 0$
$n=2$	1.5	$-.75 + 0(.25) = \boxed{-.75}$	$1 - 2(1.5) - 2(-.75) = -.5$
$n=3$	1.75	$-.75 + (-.5)(.25) = \boxed{-.875}$	