MA 214—04 Some review problems.

Here is a fairly random collection of review problems to start the term.

1. Compute some integrals:

a. $\int \frac{x^3+3x+1}{x}\,dx$ $= \int \left(\frac{x^3}{x} + \frac{3x}{x} + \frac{1}{x}\right)dx = \int \left(x^2 + 3 + \frac{1}{x}\right)dx$

$= \boxed{\frac{1}{3}x^3 + 3x + \ln|x| + C}$

b. $\int \left(e^x + \cos x + \sin x\right)dx$ $= \boxed{e^x + \sin x - \cos x + C}$

c. $\int x^2 \sin\left(x^3+4\right)dx$ $= \int x^2 \cdot \sin u \cdot \frac{1}{3x^2}\,du = \int \frac{1}{3}\sin u\,du$

$u = x^3+4$

$du = 3x^2\,dx \qquad dx = \frac{1}{3x^2}\,du$

$= -\frac{1}{3}\cos u + C$

$= \boxed{-\frac{1}{3}\cos\left(x^3+4\right) + C}$

d. $\int \frac{1}{5-2y}\,dy$

done in class

e. $\int x^2 \ln x\,dx$

done in class

2. Find a function that if we take its fourth derivative, we get 16 times the original function.

done in class

3. Solve for x: $6x^2+13x=5$. $6x^2+13x-5=0\ldots$

done in class

4. Find the radius of convergence: $\sum_{n=0}^{\infty} \frac{x^n}{3^n}$

Ratio test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{3^{n+1}}\cdot\frac{3^n}{x^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x}{3}\right| = \left|\frac{x}{3}\right| < 1$$

$-1 < \frac{x}{3} < 1$

$-3 < x < 3$

radius of convergence $= \boxed{3}$

(for interval of convergence, test endpoints)