Please write up **complete, clear solutions**. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question (summarize the directions) and write neatly.

This assignment comes from material in Chapter 1. The textbook will be a helpful reference for these. You can also get help via email (ewhitaker@uky.edu), via office hours (stop by or make an appointment) or in the Mathskeller (check the course website for specific times).

1. Find the general solution to the differential equations. Implicit solutions are fine. Hint: one is linear and the other is separable.
   a. $x\dfrac{dy}{dx}-2y=x^3e^{-2x}$
   b. $x^2y^2\dfrac{dy}{dx}+1=y$

2. Consider the differential equation $2xydx+\left(y^2-x^2\right)dy=0$.
   a. Show this equation is not exact.
   b. We can find an integrating factor $\mu=e^{-\int p(y)dy}$ if $p(y)=\dfrac{M_y-N_x}{M}$ is a function of only y. Find and apply μ, and then show that the new equation is exact.
   c. Solve the new equation. Leave your general solution in implicit form.

3. Consider the differential equation $\dfrac{dy}{dx}=\sqrt{x+y}$ with initial condition $y(1)=2$.
   a. Use Euler's method with step size $h=0.2$ to estimate $y(1.6)$.
   b. Euler's method uses the slope at (x_n,y_n) to estimate y_{n+1}. The **fourth-order Runge-Kutta method** (RK4) uses a weighted average of slopes in the interval $[x_n,x_{n+1}]$ to estimate y_{n+1}:

$$y_{n+1}=y_n+\tfrac{1}{6}\left(m_1+2m_2+2m_3+m_4\right)h$$

   where

$$\begin{aligned}
m_1 &= F\left(x_n,y_n\right)\\
m_2 &= F\left(x_n+\tfrac{h}{2},y_n+\tfrac{h}{2}m_1\right)\\
m_3 &= F\left(x_n+\tfrac{h}{2},y_n+\tfrac{h}{2}m_2\right)\\
m_4 &= F\left(x_n+h,y_n+hm_3\right)
\end{aligned}$$

   Use this method with step size $h=0.2$ to estimate $y(1.2)$.