Math 225 - Practice Midterm 1

1. True or False?

(a) The set of vectors
$$\left\{ \begin{pmatrix} 1\\0\\2 \end{pmatrix}, \begin{pmatrix} 0\\-1\\1 \end{pmatrix}, \begin{pmatrix} 0\\0\\0 \end{pmatrix} \right\}$$
 is linearly independent.
(b) The set of vectors $\left\{ \begin{pmatrix} 1\\0\\2 \end{pmatrix}, \begin{pmatrix} 0\\-1\\1 \end{pmatrix}, \begin{pmatrix} 1\\1\\1 \end{pmatrix} \right\}$ is linearly independent.
(c) The set of vectors $\left\{ \begin{pmatrix} 1\\0\\2 \end{pmatrix}, \begin{pmatrix} 0\\-1\\1 \end{pmatrix}, \begin{pmatrix} 1\\0\\0 \end{pmatrix} \right\}$ is linearly independent.

- (d) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible, then *a* cannot be 0.
- (e) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible, then *b* cannot be 0.
- (f) A homogeneous system of equations may have no solution.
- (g) If the columns of *A* are independent, then *A* must have at least as many rows as columns.
- 2. Consider the system of equations

$$w + x + y + z = 1$$

$$-w + y + z = 1$$

$$x + 2y + 3z = 1$$

- (a) Write down the corresponding augmented matrix and convert it into reduced row echelon form.
- (b) Find the general solution to this system of equations.
- (c) Give a particular solution.
- 3. Let

$$A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 3 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 5 & 2 & 1 \\ -1 & 4 & 2 \\ -2 & -2 & 3 \end{pmatrix}$$

Of the matrix products AB, BA, A^TB^T , B^TA^T , and BA^T , compute the ones that are well-defined.

4. Find the inverse of the following matrices:

(a)
$$A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$$

(b) $B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$

5. Show that if *A* is invertible, then so is A^T , with $(A^T)^{-1} = (A^{-1})^T$.