Thursday, October 20 $**$ *Echelon form, The geometry of vectors*

1. Let
$$A = \begin{pmatrix} 2 & -4 & 4 & -2 \\ 3 & 1 & -1 & -3 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$
   (a) Without switching any rows, row reduce A into echelon form. Which columns have pivots? What are the pivots?
   (b) Switch the first and third rows of A, and then row reduce the matrix into echelon form without using any additional switching of rows. Which columns have pivots? What are the pivots?
   (c) Further reduce the matrices you found in (a) and (b) into *reduced* echelon form.

2. Consider vectors $\mathbf{u}$ and $\mathbf{w}$ as represented to the right.

   (a) Draw the vectors $2\mathbf{u}$ and $-\mathbf{u}$.
   (b) Draw the vectors $\mathbf{u}+\mathbf{w}$, $\mathbf{u}-\mathbf{w}$, and $\mathbf{u}-2\mathbf{w}$.
   (c) What is the geometric description of $\mathbf{u}-\mathbf{w}$ in general?
   (d) What is $\text{Span}\{\mathbf{u},\mathbf{w}\}$?

3. Let $\mathbf{u}=(1,-1,2)$, $\mathbf{v}=(2,1,-2)$, and $\mathbf{w}=(-2,2,-4)$ in $\mathbb{R}^3$.
   (a) Describe $\text{Span}\{\mathbf{u},\mathbf{v}\}$, $\text{Span}\{\mathbf{u},\mathbf{w}\}$, and $\text{Span}\{\mathbf{u},\mathbf{v},\mathbf{w}\}$. What shape in $\mathbb{R}^3$ does each of these form?
   (b) Is the vector $\mathbf{b}=(1,0,0)$ a linear combination of $\mathbf{u}$ and $\mathbf{w}$? Is it a linear combination of $\mathbf{u}$ and $\mathbf{v}$?
   (c) Is the vector $\mathbf{c}=(1,1,1)$ a linear combination of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$?