Thursday, November 10 $\quad **\quad$ *Vector Spaces & Subspaces*

1. Let $M_2(\mathbb{R})$ be the vector space of all 2×2 matrices. In each item below, determine whether or not the given collection of vectors is a subspace of $M_2(\mathbb{R})$. If it is a subspace, find a minimal spanning set.

   (a) Let $D_2(\mathbb{R})$ be the space of all 2×2 diagonal matrices.

   (b) Let $U_2(\mathbb{R})$ be the space of all 2×2 upper triangular matrices.

   (c) Let $L_2^s(\mathbb{R})$ be the space of all 2×2 *special* lower triangular matrices.

   (d) Let $S_2(\mathbb{R})$ be the space of all 2×2 "symmetric" matrices. A matrix A is symmetric if $A = A^T$.

   (e) Let $Gl_2(\mathbb{R})$ be the space of all 2×2 invertible matrices.

2. Let $\mathcal{F}(\mathbb{R})$ be the vector space of real-valued functions of one variable. Determine whether the following subsets of $\mathcal{F}(\mathbb{R})$ are subspaces.

   (a) The collection of constant functions.

   (b) The collection of functions $f(x)$ satisfying $f(3) = 0$.

   (c) The collection of functions $f(x)$ satisfying $f(0) = 3$.

   (d) The collection of continuous functions.

   (e) The collection of bounded functions. (Recall that $f(x)$ is bounded if there is a number M such that $|f(x)| < M$ for all x.)

   (f) The collection of differentiable functions.

3. Let

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -2 & 1 & 0 & 1 \\ -4 & 1 & -2 & -1 \end{pmatrix}$$

   (a) Find a minimal spanning set for $\mathrm{Nul}(A)$.

   (b) Find a minimal spanning set for $\mathrm{Col}(A)$.