Tuesday, November 15 ** *Bases & Dimension*

1. Consider the subspace H of $\mathbb{R}^4$ defined by the equation

$$2x_1 - 3x_2 + 4x_3 + 2x_4 = 0.$$

   (a) Find a basis of H. What is the dimension of H?

   (b) Find the coordinates for the point $(2\ 2\ 1\ -1)^T$ in this basis for H.

   (c) Complete the basis you found in (a) to a basis of $\mathbb{R}^4$.

   (d) Find the coordinates for the standard basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, and $\mathbf{e}_4$ in the new basis for $\mathbb{R}^4$.

2. (a) Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a collection of vectors in the vector space V, and let $H = \text{Span}(S)$. The set S may not be linearly independent, so S is not necessarily a basis for H. Describe how to obtain a basis for H starting from the set S. Here's a hint to get you started. First, pick one vector in S, say $\mathbf{v}_1$. Next, look at the vector $\mathbf{v}_2$. If this is a multiple of $\mathbf{v}_1$, we should not include it in a basis. If $\mathbf{v}_2$ is *not* a multiple, then we include it in the basis.

   (b) Use the process in described in part (a) to find a basis for

$$\text{Span}\left\{\begin{pmatrix}1\\2\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\2\\-1\\1\end{pmatrix}, \begin{pmatrix}3\\4\\1\\-4\end{pmatrix}, \begin{pmatrix}0\\1\\4\\2\end{pmatrix}\right\}$$

3. Find a basis for and determine the dimension of the null space and column space of the following matrices

   (a) $A = \begin{pmatrix}1 & -2 & 9 & 0 & -2\\0 & 1 & 1 & -4 & 2\end{pmatrix}$

   (b) $B = \begin{pmatrix}1 & -2 & 9 & 0 & -2\\0 & 1 & 1 & -4 & 2\\0 & 0 & 0 & 1 & 2\end{pmatrix}$

   (c) $C = \begin{pmatrix}1 & -2 & 9 & 0 & -2\\0 & 1 & 1 & -4 & 2\\0 & 0 & 0 & 1 & 2\\0 & 0 & 0 & 0 & 0\end{pmatrix}$

4. The row space of a matrix is the span of the rows. Find a basis for and determine the dimension of the row space of the matrices A, B, and C from problem 3.