

Thursday, October 20 \*\* Introduction to multiple integrals

1. Evaluate the following integral by reversing the order of integration:

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} dx dy.$$

(Hint: When you change to  $dx dy$ , be sure to also change the bounds of integration.)

2. Consider the region bounded by the curves determined by  $-2x + y^2 = 6$  and  $-x + y = -1$ .

(a) Sketch the region  $R$  in the plane.

(b) Set up and evaluate an integral of the form  $\iint_R dA$  that calculates the area of  $R$ .

3. Consider the region  $R$  in the first quadrant which lies above the  $x$ -axis, to the right of the  $y$ -axis and between the circles of radius 1 and 2 centered at  $(0,0)$ . Without using polar coordinates, evaluate

$$\iint_R y dA.$$

Hint: You'll have to break  $R$  into several simple (Type I and II) regions.

4. Evaluate

$$\int_{-2}^0 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx.$$

Hint: don't do it directly.

5. The function  $P(x) = e^{-x^2}$  is fundamental in probability.

(a) Sketch the graph of  $P(x)$ . Explain why it is called a "bell curve."

(b) Compute  $I = \int_{-\infty}^{\infty} e^{-x^2} dx$  using the following brilliant strategy of Gauss.

i. Instead of computing  $I$ , compute  $I^2 = \left( \int_{-\infty}^{\infty} e^{-x^2} dx \right) \left( \int_{-\infty}^{\infty} e^{-y^2} dy \right)$ .

ii. Rewrite  $I^2$  as an integral of the form  $\iint_R f(x,y) dA$  where  $R$  is the entire Cartesian plane.

iii. Convert that integral to polar coordinates.

iv. Evaluate to find  $I^2$ . Deduce the value of  $I$ .

Amazingly, it can be mathematically proven that there is NO elementary function  $Q(x)$  (that is, function built up from sines, cosines, exponentials, and roots using "usual" operations) for which  $Q'(x) = P(x)$ .