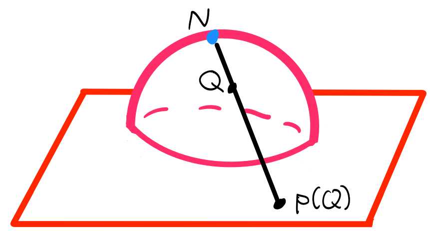
Group: ____________ ____________ ____________ Name: ____________

Math 351 - Elementary Topology
Friday, October 19 $**$ *Stereographic projection*

One way to visualize the two-sphere S^2 is as the plane $\mathbb{R}^2$ with a single point added in "at infinity". The homeomorphism that allows you to go back and forth is known as the *stereographic projection*. Let $N = (0,0,1)$ denote the north pole on the sphere.



1. Define a map $p : S^2 \setminus \{N\} \xrightarrow{\cong} \mathbb{R}^2$ as follows. For each point $Q = (x,y,z) \in S^2$, consider the straight line through N and Q. This line may be *parametrized* as

   $$N + t(Q - N).$$

   Find the unique point on this line for which $z = 0$ (solve for t). Define $p(Q)$ to be this point.

2. What are conditions on the point Q that determine whether $p(Q)$ will be inside the unit disc $B_1(\mathbf{0})$ or outside the disc?

3. Define a map $g : \mathbb{R}^2 \xrightarrow{\cong} S^2 \setminus \{N\}$ as follows. Given a point $P = (x,y) \in \mathbb{R}^2$, we may again consider the line through N and $Q = (x,y,0)$ parametrized as before. Solve for the value of t that will put this point on the sphere S^2. (Remember that (x,y,z) is on the sphere if and only if $x^2 + y^2 + z^2 = 1$.)

4. Show that the maps p and g are homeomorphisms and that $g = p^{-1}$.

Write your answer(s) on the rest of this sheet (and back).
