Group: _____ _____ _____ Name: _____

Math 351 - Elementary Topology
Wednesday, October 31 ** *Quotients – SCARY STUFF!!*

One way to form a quotient of X is to partition X into pieces, as discussed in class. A common way to do this is to put an **equivalence relation** on X. An equivalence relation on X must satisfy three rules:

- (Reflexive) For each $x \in X$, $x \sim x$.
- (Symmetric) If $x \sim y$ then $y \sim x$.
- (Transitive) If $x \sim y$ and $y \sim z$ then $x \sim z$.

The equivalence class $\overline{x}$ of $x \in X$ is the subset of X consisting of all points y that are "equivalent" to x ($y \sim x$). The equivalence classes give a partition of X. The set of equivalence classes is denoted $X/\sim$.

1. Let $X = \mathbb{R}$ and define a relation on $\mathbb{R}$ by $x \sim y$ if x and y have the same sign (or are both 0). Show this is an equivalence relation. What are the equivalence classes? What is the resulting topology?
2. Let $X = \mathbb{R}^2$ and define a relation on $\mathbb{R}$ by $(x, y) \sim (x', y')$ if

   $$x + y = x' + y'.$$

   Show this is an equivalence relation and find the equivalence classes. What is the resulting space $\mathbb{R}^2/\sim$?

Write your answer(s) on the rest of this sheet (and back).
