Math 551 - Topology I
Homework 2
Fall 2013

1. Let (X, d) be a metric space and let $x \in X$. Define $d_x : X \longrightarrow \mathbb{R}$ by $d_x(y) = d(x, y)$. Show that d_x is continuous ($\mathbb{R}$ has the usual metric).

2. In the definition of topological space, we ask only for *finite* intersections of open sets to be open. Give an example of an infinite intersection of open sets which is no longer open.

3. (a) (The cofinite topology) Let X be an infinite set. Define a nonempty subset $U \subseteq X$ to be open if $X \setminus U$ is finite. Show that this defines a topology on X.
   (b) (The cocountable topology) Let X be an infinite set. Define a nonempty subset $U \subseteq X$ to be open if $X \setminus U$ is countable. Show that this defines a topology on X.
   (c) In the case $X = \mathbb{R}$, how do these relate to each other and to the usual topology ?

4. (a) (Generic point topology) Let X be a set, and fix a special point $x_0 \in X$. Declare a nonempty subset $U \subseteq X$ to be open in X if and only if $x_0 \in U$. Show that this gives a topology on X.
   (b) (Excluded point topology) Let X be a set, and fix a special point $x_0 \in X$. Declare a proper subset $U \subset X$ to be open if and only if $x_0 \notin U$. Show that this gives a topology on X.

5. Show that if Y is a set equipped with the trivial topology and X is any space, then every function $f : X \longrightarrow Y$ is continuous.

6. Find all topologies on the set $X = \{0, 1, 2\}$.

7. (Vertical interval topology) Define $\mathcal{B}$ to be the collection of vertical open intervals in $\mathbb{R}^2$, namely, sets of the form
$$\{(a, x) \mid b < x < c\}$$
for fixed a, b, c. Show that this defines a basis for a topology on $\mathbb{R}^2$. How is this topology related to the standard topology?