

Math 551 - Topology I
Homework 3
Fall 2013

1. Let X be a space and $A \subseteq X$ a subset. Show that $\overline{X \setminus A} = X \setminus \text{Int } A$, and use this to show that ∂A is always closed.

2. Let $A, B \subseteq X$.
 - (a) Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
 - (b) Show that $\bigcup_i \overline{A_i} \subseteq \overline{\bigcup_i A_i}$. Give an example to show that equality need not hold in general.

3. Let $X = \mathbb{R}_{\text{cofinite}}$ be the real line equipped with the cofinite topology.
 - (a) Show that if $\{x_n\}$ is a sequence *with no repeated terms*, then $\{x_n\}$ converges to **every real number**.
 - (b) Show that the alternating sequence $(1, -1, 1, -1, \dots)$ does not converge.
 - (c) Show that the sequence $(1, 2, 1, 3, 1, 4, 1, 5, \dots)$ converges to 1.

4. Let $X = \mathbb{R}_{\ell\ell}$ be the real line equipped with the lower limit topology. Recall that this has basis given by the $[a, b)$.
 - (a) Show that $(1/n) \rightarrow 0$.
 - (b) Show that $(\frac{n-1}{n})$ does not converge. (You may find it useful to note that the identity map $\text{id} : \mathbb{R}_{\ell\ell} \rightarrow \mathbb{R}_{\text{standard}}$ is continuous.)

5. (The line with doubled origin) Let $X = \mathbb{R} \cup \{0'\}$ with topology as follows: a subset $U \subseteq X$ is open if either $U \subseteq \mathbb{R} \setminus \{0\}$ and U is open in the usual topology on \mathbb{R} , or $\{0, 0'\} \subseteq U$ and $U \cap \mathbb{R}$ is open in \mathbb{R} . Show that the sequence $(1/n)$ converges to **both** 0 and $0'$.

6. Find the closure of $[0, 1)$ in $\mathbb{R}_{\ell\ell}$, the lower limit topology.