# Math 551 - Topology I
# Homework 4
# Fall 2013

1. Consider the topology on $\mathbb{R}$ given by the subbasis consisting of open intervals $(a, \infty)$.
   (a) Given a subset $A \subseteq \mathbb{R}$, describe the closure $\overline{A}$ in this topology.
   (b) Consider the sequence $x_n = n$. Does it converge? If so, to what?
   (c) Show that the sequence lemma holds in this topology. That is, if $x \in \overline{A}$, then some sequence in $A$ converges to $x$.

   This is an example of a non-Hausdorff space in which the sequence lemma holds.

2. (a) Show that for any two spaces $X$ and $Y$, the projection map $p_X : X \times Y \longrightarrow X$ is *open*.
   (b) Find an example to show that the projection $p_X$ need not be closed.

3. Show that a space $X$ is Hausdorff if and only if the diagonal subset

$$\Delta(X) = \{(x, x)\} \subseteq X \times X$$

   is closed.

4. Recall that if $I$ is a set and we have $X_i = X$ for each $i \in I$, then the product $\prod_i X_i$ can be identified with the collection of functions $f : I \longrightarrow X$. Consider this as a space with the product topology. Show that $(f_n) \to f$ in this topology if and only if the functions converge to $f$ *pointwise*.

5. Let $\mathcal{Z} \subseteq \mathbb{R}^{\mathbb{N}}$ be the subset consisting of sequences which are eventually zero (in other words, only finitely many of the entries are nonzero). Find the closure of $\mathcal{Z}$ in $\mathbb{R}^{\mathbb{N}}$ under the box and product topologies

6. Let $A_i \subseteq X_i$ be a subspace for each $i \in I$. Show that the subspace topology on

$$\prod_{i \in I} A_i \subseteq \prod_{i \in I} X_i$$

   coincides with the product topology (and therefore not, in general, with the box topology)