Math 551 - Topology I
Homework 6
Fall 2013

1. (Topologist's sine curve) Let $\Gamma \subseteq \mathbb{R}^2$ be the graph of $\sin(1/x)$ for $0 < x \leq 1/\pi$. Show that the closure $\overline{\Gamma}$ is connected but not path-connected, locally connected, or locally path-connected.

2. Show that $\mathbb{R}_{\text{cocountable}}$ is connected and locally connected but not path-connected or locally path-connected.

3. For any space X, define the **cone** on X to be
$$CX = (X \times I)/(X \times \{1\}).$$
Show that CX is path-connected (no assumptions on X).

4. Show that $\mathbb{R}_{\text{cofinite}}$ is compact.

5. Show that if X is a metric space and $A \subseteq X$ is compact, then A is closed and bounded (contained in a single ball of finite radius).

6. (a) Show that if X is compact and Z is any space, then the projection $p_Z : X \times Z \longrightarrow Z$ is closed.

   (b) $(\star)$ Show that the converse to (a) also holds. That is, if p_Z is closed for every Z, then X is compact.