Math 114 - Calculus II
Tuesday, November 14 ** *Quiz 5* **SOLUTIONS**

1. Determine whether the following series either converge absolutely (A), converge conditionally (C), or diverge (D). Make sure to state clearly what test(s) you are using.

   (a) $\sum_{n=1}^{\infty} \frac{n^2+1}{2n^2+3n+1}$

   We have
   $$\lim_n \frac{n^2+1}{2n^2+3n+1} = \frac{1}{2} \neq 0,$$
   so the series diverges by the divergence test.

   A C D (circle one answer)

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+1}$

   This is an alternating series. Since $\frac{1}{1+1} > \frac{1}{\sqrt{2}+1} > \frac{1}{\sqrt{3}+1} > \dots$ and since $\lim_n \frac{1}{\sqrt{n}+1} = 0$, we conclude that the series converges by the Alternating Series Test.
   To see that it does not converge absolutely, we use a Limit Comparison Test, comparing the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$ to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$. This second series diverges since $p = \frac{1}{2} \leq 1$.
   $$\lim_n \frac{\frac{1}{\sqrt{n}+1}}{\frac{1}{\sqrt{n}}} = \lim_n \sqrt{\frac{n}{n+1}} = \sqrt{\lim_n \frac{n}{n+1}} = \sqrt{1} = 1.$$
   Since this limit is > 0, it follows that our series behaves the same as the p-series. Since the p-series diverges, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$ also diverges. It follows that our orignial series converges conditionally.

   A C D (circle one answer)

   (c) $\sum_{n=2}^{\infty} (-1)^{n^2+1} \frac{5^n}{(n+1)!}$

   We use the Ratio Test.
   $$\lim_n \frac{5^{n+1}/(n+2)!}{5^n/(n+1)!} = \lim_n \frac{5^{n+1}}{5^n} \frac{(n+1)!}{(n+2)!} = \lim_n \frac{5}{n+2} = 0.$$
   Since this is less than 1, our series converges absolutely.

   A C D (circle one answer)

(**CONTINUED ON BACK**)

2. (a) Use the (Leibniz) Alternating Series Test to show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges. Make sure to check all of the hypotheses!

Let $a_n = \frac{1}{n^2}$. In order to apply the Alternating Series Test, we need to check that a_n is decreasing towards 0. We have $\lim_n \frac{1}{n^2} = 0$ as needed. To check that it is decreasing, we need to show that $\frac{1}{n^2} > \frac{1}{(n+1)^2}$. This is equivalent to showing that $(n+1)^2 > n^2$. But

$$(n+1)^2 = n^2 + 2n + 1 \geq n^2 + 1 > n^2,$$

so we are done. Another way to do this would be to show that the derivative of $1/x^2$ is negative. It now follows from the Alternating Series Test that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges.

(b) How many terms are needed in order to guarantee that the partial sum will be within $\frac{1}{1000}$ of the infinite sum?

The Alternating Series Test further tells us that if $S = \sum_{n=1}^{\infty} (-1)^n a_n$ and $S_N = \sum_{n=1}^{N} (-1)^n a_n$ with all $a_n > 0$, then

$$|S - S_N| < a_{N+1}.$$

Since we want to make $|S - S_N|$ less than $\frac{1}{1000}$, it suffices to find an N such that $a_{N+1} < \frac{1}{1000}$. But $A_{N+1} = \frac{1}{(N+1)^2}$. Solving

$$\frac{1}{(N+1)^2} < \frac{1}{1000}$$

gives $N + 1 > \sqrt{1000} = 10\sqrt{10} \approx 31.6$. Thus we want $N + 1 \geq 32$ or $N \geq 31$.