Math 551 - Topology I
Homework 10
Fall 2014

1. Fill in the details to show that $\mathbb{RP}^n$ is an n-manifold.

2. Recall that for any $P \in S^1$, the punctured circle $U_P = S^1 \setminus \{P\}$ is homeomorphic to $\mathbb{R}^1$.

   (a) Pick two points $P, Q \in S^1$ and cover S^1 by U_P and U_Q. Find a partition of unity subordinate to this cover.

   (b) Following the embedding theorem from class, this gives an embedding $E : S^1 \hookrightarrow \mathbb{R}^4$. Describe the map $S^1 \longrightarrow \mathbb{R}^2$ obtained by using the first two coordinates of this embedding. Following notation from the proof of the embedding theorem, this would be the map

   $$(f_1\varphi_1, f_2\varphi_2) : S^1 \longrightarrow \mathbb{R}^2.$$

   In particular, you should show that this is *not* an embedding.

3. (a) Show that if A is discrete, then $X \times A \cong \coprod_A X$.

   (b) Show that if X and Y are any spaces and A is discrete, then there is a bijection

   $$\mathcal{C}(X \times A, Y) \cong \mathcal{C}(X, Y^A)$$

   if Y^A is given the product topology.

4. (a) Suppose that Y is locally compact Hausdorff. Let $K \subseteq U \subset Y$ with K compact and U open. Show that there is a precompact open set V with

   $$K \subseteq V \subseteq \overline{V} \subseteq U.$$

   (b) Let Y be locally compact Hausdorff and X and Z be arbitrary. Show that the "composition" map

   $$\mathrm{Map}(Y, Z) \times \mathrm{Map}(X, Y) \longrightarrow \mathrm{Map}(X, Z)$$

   defined by $(g, f) \mapsto gf$ is continuous.