Math 551 - Topology I
Homework 3
Fall 2014

1. Let X be a space and $A \subseteq X$ a subset. Show that $\overline{X \setminus A} = X \setminus \operatorname{Int} A$, and use this to show that ∂A is always closed.

2. Let $A, B \subseteq X$.

   (a) Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

   (b) Show that $\bigcup_i \overline{A_i} \subset \overline{\bigcup_i A_i}$. Give an example to show that equality need not hold in general.

3. Let $X = \mathbb{R}_{\text{cofinite}}$ be the real line equipped with the cofinite topology.

   (a) Show that if $\{x_n\}$ is a sequence *with no repeated terms*, then $\{x_n\}$ converges to **every real number**.

   (b) Show that the alternating sequence $(1, -1, 1, -1, \ldots)$ does not converge.

   (c) Show that the sequence (1,2,1,3,1,4,1,5,...) converges to 1.

4. Let $X = \mathbb{R}_{\ell\ell}$ be the real line equipped with the lower limit topology. Recall that this has basis given by the $[a, b)$.

   (a) Show that $(1/n) \to 0$.

   (b) Show that $(\frac{n-1}{n})$ does not converge. (You may find it useful to note that the identity map $\mathrm{id} : \mathbb{R}_{\ell\ell} \to \mathbb{R}_{\text{standard}}$ is continuous.)

5. (The line with doubled origin) Let $X = \mathbb{R} \cup \{0'\}$ with topology as follows: a subset $U \subseteq \mathbb{R}$ is open if it is open in the usual topology on $\mathbb{R}$. For a subset $V \subseteq X$ that contains the new origin $0'$, we declare it to be open if $(V - \{0'\}) \cup \{0\}$ is open in $\mathbb{R}$. That is, we replace $0'$ by 0 and ask if that is open in the usual sense. Show that the sequence $(1/n)$ converges to **both** 0 and $0'$.