Math 551 - Topology I
Homework 4
Fall 2014

1. Consider the topology on $\mathbb{R}$ given by the subbasis consisting of open intervals (a, ∞).
   (a) Given a subset $A \subseteq \mathbb{R}$, describe the closure $\overline{A}$ in this topology.
   (b) Consider the sequence $x_n = n$. Does it converge? If so, to what?
   (c) Show that the sequence lemma holds in this topology. That is, if $x \in \overline{A}$, then some sequence in A converges to x.

   This is an example of a non-Hausdorff space in which the sequence lemma holds.

2. Consider the generic point topology on a set X, with generic point $p \in X$.
   (a) To which point(s) in X does the constant sequence $x_n = p$ converge?
   (b) Now let x_n be a sequence in X which avoids, p, that is, $x_n \neq p$ for all n. To which point(s) in X does x_n converge?

3. Let X be Hausdorff, let $A \subseteq X$, and let A' denote the set of accumulation points of A. Show that A' is closed in X.

4. (a) Show that for any two spaces X and Y, the projection map $p_X : X \times Y \longrightarrow X$ is *open*.
   (b) Find an example to show that the projection p_X need not be closed.

5. Show that a space X is Hausdorff if and only if the diagonal subset

$$\Delta(X) = \{(x, x)\} \subseteq X \times X$$

   is closed.