Math 551 - Topology I
Homework 5
Fall 2014

1. Recall that if A is a set and we have $X_\alpha = X$ for each $\alpha \in A$, then the product $\prod_\alpha X_\alpha$ can be identified with the collection of functions $f : A \longrightarrow X$. Consider this as a space with the product topology. Show that $(f_n) \to f$ in this topology if and only if the functions converge to f *pointwise*.

2. Let $\mathcal{Z} \subseteq \mathbb{R}^{\mathbb{N}}$ be the subset consisting of sequences which are eventually zero (in other words, only finitely many of the entries are nonzero). Find the closure of $\mathcal{Z}$ in $\mathbb{R}^{\mathbb{N}}$ under the box and product topologies

3. Let $A_j \subseteq X_j$ be a subspace for each $j \in J$. Show that the subspace topology on

$$\prod_{j \in J} A_j \subseteq \prod_{j \in J} X_j$$

coincides with the product topology (here $\prod_{j \in J} X_j$ is equipped with the product topology).

4. Show that $\mathbb{R}^2$, equipped with the *vertical interval* topology (HW 2-6), is isomorphic to $\mathbb{R}_{\text{discrete}} \times \mathbb{R}$.

5. A **based space** is simply a space X with a chosen basepoint $x_0 \in X$. We say that a map $f : X \longrightarrow Y$ is based (or basepoint-preserving) if $f(x_0) = y_0$, where x_0 and y_0 are the chosen basepoints of X and Y, respectively.

   Let (X, x_0) and (Y, y_0) be based spaces. We define their **wedge sum** to be

$$X \vee Y = X \coprod Y / \sim$$

   where $\iota_X(x_0) \sim \iota_Y(y_0)$.

   (a) Show that the wedge sum satisfies the universal property of the coproduct for based spaces. That is, if $f : (X, x_0) \longrightarrow (Z, z_0)$ and $g : (Y, y_0) \longrightarrow (Z, z_0)$ are based maps, there is a unique continuous map $h : (X \vee Y, \overline{x_0}) \longrightarrow (Z, z_0)$ which makes the appropriate diagram commute.

   (b) Show that if X and Y are Hausdorff based spaces, then so is $X \vee Y$.