## Math 551 - Topology I Homework 6 Fall 2014

- 1. (Topologist's sine curve) Let  $\Gamma \subseteq \mathbb{R}^2$  be the graph of  $\sin(1/x)$  for  $0 < x \leq 1/\pi$ . Show that the closure  $\overline{\Gamma}$  is connected but not path-connected, locally connected, or locally path-connected.
- 2. For any space *X*, define the **cone** on *X* to be

 $CX = (X \times I) / (X \times \{1\}).$ 

Show that *CX* is path-connected (no assumptions on *X*).

- 3. Show that  $\mathbb{R}_{\text{cocountable}}$  is connected and locally connected but not path-connected or locally path-connected. (Hint: For determining path-connectedness, it may help to first determine what are the *compact* subsets of  $\mathbb{R}_{\text{coco}}$ )
- 4. Show that  $\mathbb{R}_{\text{cofinite}}$  is compact.
- 5. (a) Show that if *X* is compact and *Z* is any space, then the projection  $p_Z : X \times Z \longrightarrow Z$  is closed.

(b) ( $\star$ ) Show that the converse to (a) also holds. That is, if  $p_Z$  is closed for every *Z*, then *X* is compact.