Math 551 - Topology I
Homework 9
Fall 2014

1. A map $f : X \longrightarrow Y$ is said to be **proper** if, for any compact subset $K \subseteq Y$, the preimage $f^{-1}(K) \subseteq X$ is compact.

   (a) Show that if X is compact and Y is Hausdorff, then any continuous $f : X \longrightarrow Y$ is automatically proper.

   (b) Let X and Y be locally compact and Hausdorff. Show that a continuous map $f : X \longrightarrow Y$ is proper if and only if it extends to a continuous map $\widehat{f} : \widehat{X} \longrightarrow \widehat{Y}$ with $\widehat{f}(\infty_X) = \infty_Y$.

2. Show that if X is normal and $A \subseteq X$ is closed, then A is normal.

3. Suppose that X is normal and connected. Show that if X contains more than one point, it must be uncountable. (Hint: Use the Urysohn Lemma.)