

Math 654 - Algebraic Topology

Homework 2

Fall 2015

1. Given the Δ -complex structure on the Klein bottle K described in class, compute the homology groups $H_*^\Delta(K)$.
2. Let X be obtained from a simplex Δ^2 by identifying the three vertices to a single point. Compute the homology groups $H_*^\Delta(X)$.
3. Build S^3 as a Δ -complex, and use this to compute $H_*^\Delta(S^3)$.
4. Consider a ball B^3 , obtained by gluing together three Δ^3 's along the edge $\{2, 3\}$ as in the picture to the right. Let X be the space obtained by the gluings $e_1 \sim f_2$, $e_2 \sim f_3$, and $e_3 \sim f_1$. In the picture, e_3 and f_3 are the "back" faces. Compute $H_*(X)$.
(Hint: you should find that $H_1 \cong \mathbb{Z}/3$.)

