

# Math 654 - Algebraic Topology

## Homework 3

### Fall 2015

1. Recall that the **cone**  $CX$  on a space  $X$  is defined by

$$CX = (X \times I) / (X \times 1).$$

Using the fact that  $C\Delta^n \cong \Delta^{n+1}$ , convince yourself that if  $X$  is a  $\Delta$ -complex, then  $CX$  inherits the structure of a  $\Delta$ -complex.

- (a) If  $A$  is a finite set, compute  $H_*^\Delta(CA)$ .
- (b) Compute  $H_*^\Delta(CS^1)$ .
- (c) Compute  $H_*^\Delta(CS^2)$ .
- (d) Compute  $H_*^\Delta(CT^2)$ .

What do you expect the answer to be in general?

2. (Reduced homology)

- (a) What are the reduced (simplicial) homology groups of  $S^1$ ,  $S^2$ , and  $S^3$ ?  
Recall that the (unreduced) suspension of  $X$  is  $SX = CX \cup_X CX$ .
- (b) What are the groups  $\tilde{H}_*^\Delta(ST^2)$ ?

In general, how do you expect  $\tilde{H}_*^\Delta(SX)$  to be related to  $\tilde{H}_*^\Delta(X)$ ?

3. If  $X$  and  $Y$  are  $\Delta$ -complexes with basepoints given by a choice of 0-simplex, then the wedge  $X \vee Y$  inherits a  $\Delta$ -complex structure.

- (a) Find the reduced homology groups  $\tilde{H}_*^\Delta(S^1 \vee S^1)$ .
- (b) Find the reduced homology groups  $\tilde{H}_*^\Delta(S^1 \vee S^2)$ .