Math 654 - Algebraic Topology
Homework 6
Fall 2015

1. Use the long exact sequence and excision to compute $\mathrm{H}_*(T^2)$, assuming $\mathrm{H}_1(T^2) \cong \mathbb{Z} \oplus \mathbb{Z}$.

2. Show that if M is a surface, then the only possible values for $\mathrm{H}_2(M)$ are 0 and $\mathbb{Z}$. Further show that if M is orientable of genus g, then $\mathrm{H}_1(M)$ can be generated by $2g$ elements, while if M is nonorientable, then $\mathrm{H}_1(M)$ can be generated by g elements.

3. $\mathbb{RP}^3$ can be built from $\mathbb{RP}^2$ by attaching a single 3-cell. If x denotes a point in the interior of the 3-cell, then $\mathbb{RP}^3 - \{x\} \simeq \mathbb{RP}^2$. Use the long exact sequence and excision to compute $\mathrm{H}_*(\mathbb{RP}^3)$.

4. $\mathbb{CP}^n$ can be built from $\mathbb{CP}^{n-1}$ by attaching a $2n$-cell. (Recall that $\mathbb{CP}^1 \cong S^2$.) If x denotes a point in the interior of the $2n$-cell, then $\mathbb{CP}^n - \{x\} \simeq \mathbb{CP}^{n-1}$. Use the long exact sequence and excision to compute $\mathrm{H}_*(\mathbb{CP}^n)$.