Math 322 - Matrix Algebra SOLUTIONS

Friday, September 11 $**$ *Quiz 2* Name: ____________________

1. True/False. No justification required.

   (a) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, then Span$\{\mathbf{u}, \mathbf{v}\}$ is always a **plane** through the origin. T / F

   The span could also be a line or just a point.

   (b) If the columns of A span $\mathbb{R}^n$, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for all vectors $\mathbf{b}$ in $\mathbb{R}^n$. T / F

   The span is the set of linear combinations of columns, so by assumption any $\mathbf{b}$ can be written as a combination of the columns of A. This means that $A\mathbf{x} = \mathbf{b}$ has a solution.

   (c) The matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $(A \mid \mathbf{b})$ has a pivot in each row. T / F

   For example, $(A \mid \mathbf{b}) = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$ has a pivot in each row but is inconsistent. In general, the augmented matrix is consistent if there is no pivot in the augmentation column.

2. Express the vector $\begin{pmatrix} 2 \\ -6 \\ -5 \end{pmatrix}$ as a linear combination of the vectors $\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$.

We just need to solve the matrix equation $\begin{pmatrix} -2 & -2 \\ 3 & 5 \\ 2 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ -6 \\ -5 \end{pmatrix}$.

$$\left(\begin{array}{cc|c} -2 & -2 & 2 \\ 3 & 5 & -6 \\ 2 & 4 & -5 \end{array}\right) \underset{R_3+R_1}{\overset{R_2+R_1}{\sim}} \left(\begin{array}{cc|c} -2 & -2 & 2 \\ 1 & 3 & -4 \\ 0 & 2 & -3 \end{array}\right) \overset{R_1 \leftrightarrow R_2}{\sim} \left(\begin{array}{cc|c} 1 & 3 & -4 \\ -2 & -2 & 2 \\ 0 & 2 & -3 \end{array}\right)$$

$$\overset{R_2+2R_1}{\sim} \left(\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 4 & -6 \\ 0 & 2 & -3 \end{array}\right) \overset{R_2 \leftrightarrow R_3}{\sim} \left(\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 2 & -3 \\ 0 & 4 & -6 \end{array}\right)$$

$$\overset{R_3-2R_2}{\sim} \left(\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \end{array}\right) \overset{\frac{1}{2}R_2}{\sim} \left(\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 1 & \frac{-3}{2} \\ 0 & 0 & 0 \end{array}\right)$$

$$\overset{R_1-3R_2}{\sim} \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{-3}{2} \\ 0 & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix} 2 \\ -6 \\ -5 \end{pmatrix} = \boxed{1/2} \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} + \boxed{-3/2} \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$$