Math 322 - Matrix Algebra SOLUTIONS

Friday, September 18 ∗∗ *Quiz 3* Name: ____________________

1. (**3 points**) Find the general solution to the **homogenous** system corresponding to the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix}$.

We have
$$\begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix} \underset{R_3-2R_1}{\overset{R_2+R_1}{\sim}} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & -2 & -6 \end{pmatrix} \underset{R_3+R_2}{\sim} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & -0 \end{pmatrix}$$
$$\underset{\frac{1}{2}R_2}{\sim} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & -0 \end{pmatrix} \underset{R_1-2R_2}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & -0 \end{pmatrix}$$

So x_3 is free, and the general solution is $\mathbf{x} = x_3 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

2. True/False. No justification required. (**2 points each**)

(a) It is not possible for three vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^4$ to be linearly independent. T / F

The vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are independent.

(b) If A is a 2×3 matrix, then $T(\mathbf{x}) = A\mathbf{x}$ defines a linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ T / F

3. (**3 points**) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^2$, and let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be a linear transformation such that $T(\mathbf{u}) = \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix}$ and $T(\mathbf{v}) = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$. Find the following vectors:

(a) $T(2\mathbf{u} - \mathbf{v})$

$$T(2\mathbf{u} - \mathbf{v}) = 2T(\mathbf{u}) - T(\mathbf{v}) = 2\begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \\ 11 \end{pmatrix}$$

(b) $T\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)$

Like any linear transformation, T must transform a zero vector to a zero vector, so $T\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.