**Math 322 - Matrix Algebra** SOLUTIONS

**Friday, September 25** ** *Quiz 4* Name: ____________________

1. (**4 points**) Suppose that

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 2 \\ 3 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 0 & 5 \end{pmatrix}.$$

Find the matrix products $AB$ and $BA$, if they are defined. If either is not defined, simply write **NOT DEFINED**.

Both products are defined. Since $A$ is $3 \times 2$ and $B$ is $2 \times 3$, the products $AB$ and $BA$ will be of shape $3 \times 3$ and $2 \times 2$, respectively.

$$AB = \begin{pmatrix} 2 & 8 & 17 \\ -6 & -4 & 4 \\ 1 & 6 & 14 \end{pmatrix} \qquad BA = \begin{pmatrix} 9 & 8 \\ 7 & 3 \end{pmatrix}$$

2. (**2 points**) True/False. No justification required.

If the matrix product $AB$ is defined, then the entry in row 2 and column 3 of $AB$ can be computed by pairing row 2 of $A$ with column 3 of $B$. (T) / F

3. (**4 points**)

(a) Let $T_1 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation which rotates a vector counterclockwise by 180°. Find the *standard matrix* $A_1$ corresponding to this transformation.

We discussed a general rotation matrix in class, and this has form

$$A = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}.$$

Plugging in $\theta = 180°$ gives

$$A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(b) Let $T_2 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation which **first** rotates a vector counterclockwise by 180° and **next** reflects across the (vertical) $y$-axis. Find the *standard matrix* $A_2$ corresponding to this transformation.

Composing transformations corresponds to multiplying matrices, so we can just multiply our matrix from (a) by the reflection matrix, which is $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Since the rotation happens first and the reflection second, we put the matrix $A_1$ on the *right* (so that it interacts with a vector first) and $B$ on the left (so that it meets the vector second).

$$A_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$