**Math 322 - Matrix Algebra** SOLUTIONS
**Friday, October 23** ** *Quiz 7* Name: _____

1. (**4 points**) Find a basis for the null space and the column space of $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 6 \\ -1 & 0 & 0 \end{pmatrix}$.

$$A \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

There are two pivots and one free variable, so $\dim(N(A)) = 1$ and $\dim(C(A)) = 2$.

basis for $N(A) = \left\{ \begin{pmatrix} 0 \\ -\frac{3}{2} \\ 1 \end{pmatrix} \right\}$

basis for $C(A) = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} \right\}$

2. (**6 points**) Find the dimensions of the following subspaces:

(a) $H = \{(x,y) \mid x + y = 0\} \subseteq \mathbb{R}^2$

$\dim H = 1$

(b) $H = \text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \subseteq \mathbb{R}^3$

$\dim H = 3$

(c) $H = \{(x,y,z) \mid x + y + z = 0\} \subseteq \mathbb{R}^3$

$\dim H = 2$

(d) $H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid c = 0 \right\} \subseteq M_{2\times 2}$

$\dim H = 3$

(e) $\mathbb{P}_3$ = polynomials of degree less than or equal to 3

$\dim \mathbb{P}_3 = 4$

(f) $H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a + d = 0 \right\} \subseteq M_{2\times 2}$

$\dim H = 3$