Math 322 - Matrix Algebra SOLUTIONS

Friday, November 6 ∗∗ *Quiz 8* Name: ____________________

1. (**4 points**) The matrix $A = \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{pmatrix}$ has eigenvalues 1 and 5. Find a basis for the eigenspaces E_1 and E_5.

$$\lambda = 1: \qquad A - 1I = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So we get eigenvectors $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

$$\lambda = 5: \qquad A - 5I = \begin{pmatrix} -3 & 2 & -1 \\ 1 & -2 & -1 \\ -1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -1 \\ -3 & 2 & -1 \\ -1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -1 \\ 0 & -4 & -4 \\ 0 & -4 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

So we get an eigenvector $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$.

$$\boxed{\text{basis for } E_1 = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}} \qquad \boxed{\text{basis for } E_5 = \left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}}$$

2. (**2 points each**) True/False. No justification required.

   (a) If **u** and **v** are both eigenvectors of A, then so is $\mathbf{u} + \mathbf{v}$. T / F

   This is only true if they are eigenvectors for the same eigenvalue.

   (b) If $\lambda = 0$ is an eigenvalue of A, then A cannot be invertible. T / F

   The null space of A is nontrivial, so it cannot be invertible.

3. (**2 points**) Suppose that **v** is an eigenvector of A with eigenvalue 5. Show that **v** is also an eigenvector of A^2, and find the corresponding eigenvalue.

$A^2\mathbf{v} = A(A(\mathbf{v})) = A(5\mathbf{v}) = 5A\mathbf{v} = 5 \cdot 5\mathbf{v} = 25\mathbf{v}$.

$$\boxed{\lambda = 25}$$