Math 322 - Matrix Algebra SOLUTIONS
Friday, November 13 ∗∗ *Quiz 9* Name: ____________________

1. (**2 points each**) Find the characteristic polynomial and any eigenvalues of the following matrices

(a) $A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$

$$\begin{aligned} \mathbf{p}(\lambda) &= \lambda^2 - \text{tr}(A)\lambda + \det(A) = \lambda^2 - (4+4)\lambda + (4^2 - 1^2) \\ &= \lambda^2 - 8\lambda + 15 = (\lambda - 3)(\lambda - 5) \end{aligned}$$

$\boxed{\mathbf{p}(\lambda) = \lambda^2 - 8\lambda + 15}$ $\boxed{\lambda = 3,\ 5}$

(b) $B = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}$

$$\begin{aligned} \mathbf{p}(\lambda) &= \lambda^2 - \text{tr}(A)\lambda + \det(A) = \lambda^2 - (2+1)\lambda + (2*1 - 2*3) \\ &= \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1) \end{aligned}$$

$\boxed{\mathbf{p}(\lambda) = \lambda^2 - 3\lambda - 4}$ $\boxed{\lambda = 4,\ -1}$

2. (**2 points each**) True/False. No justification required.

(a) If A is 2×2 with two distinct eigenvalues, then A is diagonalizable. T / F

If A is $n \times n$ and has n distinct eigenvalues, then it is always diagonalizable.

(b) If A is invertible and diagonalizable, then A^{-1} is also diagonalizable. T / F

If $A = CDC^{-1}$, then $A^{-1} = (CDC^{-1})^{-1} = (C^{-1})^{-1}D^{-1}C^{-1} = CD^{-1}C^{-1}$. Since D^{-1} is also a diagonal matrix, this gives a diagonalization of A^{-1}.

3. (**2 points**) Give an example of an invertible 2×2 matrix that is **not** diagonalizable. (Justify your answer.)

One answer would be the rotation matrix $A = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$. The characteristic polynomial $\lambda^2 + 1$ does not factor, so there are no (real) eigenvalues.

Another answer would be $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$. This has eigenvalue $\lambda = 2$, but only one eigenvector, so A is not diagonalizable.