

1. (1 point each) Let $\mathbf{u} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$, and $\mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. Find the following quantities

(a) $\mathbf{u} \cdot \mathbf{v} =$

12

(b) $\mathbf{u} \cdot \mathbf{w} =$

0

(c) $\mathbf{v} \cdot \mathbf{w} =$

8

(d) $\|\mathbf{u}\| =$

$\sqrt{14}$

2. (3 points) Find a basis for the plane in \mathbb{R}^3 perpendicular to $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

The plane perpendicular to this vector is the same as the null space of $(1 \ 2 \ 3)$, which is

already in reduced echelon form. We therefore read off the basis

$$\mathcal{B} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

3. (3 points) Find a nonzero vector in \mathbb{R}^3 perpendicular to the plane $W = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} \right\}$.

Again, this is just a nonzero vector in the null space of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & -5 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & -\frac{4}{5} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} \end{pmatrix}.$$

$$\mathbf{v} = \begin{pmatrix} 1/5 \\ 4/5 \\ 1 \end{pmatrix}$$